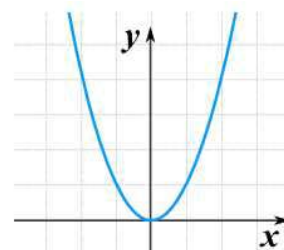


## Extra Practice/Review of Transformations of Graphs

### Transformation Review

In Common Core Math 1 and our Unit 2, we studied basic transformations of graphs, which included horizontal and vertical shifts. In this lesson, we want to review this information and expand your knowledge to include stretching and compressing graphs. **Follow along with your graphing calculator. You may want to keep the parent function in the  $y =$  screen to compare it to the transformed graph.**

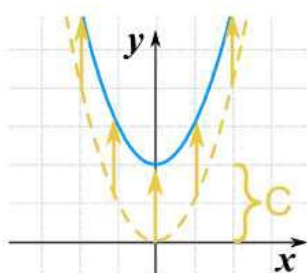


$$f(x) = x^2$$

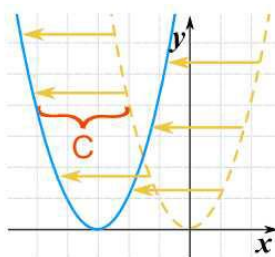
We'll begin with a basic quadratic function,  $f(x) = x^2$ , as shown below. A **vertical** shift will occur when you add a constant  $c$  to the  $y$ -value of the original function. In fact, if you add  $c$  to the graph and  $c > 0$ , the function moves up and if  $c < 0$ , the function moves down.

A **horizontal** shift will occur when you add a constant to the  $x$ -value of the graph. If you add a constant  $c$  to the  $x$ -value and  $c > 0$ , the graph moves to the left and if  $c < 0$ , the graph moves to the right. What?? Isn't that backwards??? Well no, not really. Think of it this way: Imagine you will inherit a fortune when you turn 30 (age = 30). If you change this to  $(\text{age} + 4) = 30$ , then you will get the fortune at age 26, which is 4 years earlier. Thus, adding 4 made the inheritance occur earlier.

So the moral to the story:



$$g(x) = x^2 + C$$



$$g(x) = (x+C)^2$$

$C > 0$ , shift left  
 $C < 0$ , shift right

#### Check for understanding:

How are the following graphs changed from their parent graph?

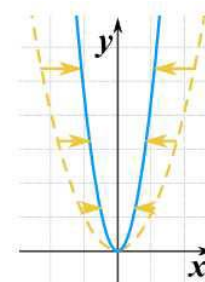
1)  $y = (x - 5)^2$  (parent:  $y = x^2$ )

2)  $y = x^2 - 5$  (parent:  $y = x^2$ )

3)  $y = x^2 + 3$  (parent:  $y = x^2$ )

4)  $y = (x + 4)^2$  (parent:  $y = x^2$ )

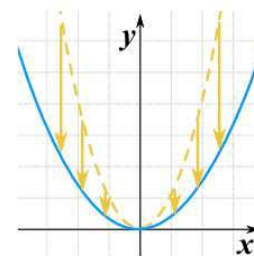
Now for something different! You can stretch or compress the function in the  **$y$  direction** by multiplying the entire function by a constant  $c$ , so that you have  $cf(x)$ . In this case, **stretching** the function means that the graph of the function moves closer to the  $y$ -axis and **compressing** the function means that the graph of the function moves closer to the  $x$ -axis. If  $c > 1$ , the graph stretches. If  $0 < c < 1$ , the graph compresses.



$$g(x) = (2x)^2$$

You can also stretch or compress the graph in the  **$x$  direction** by multiplying  $x$  by a constant, so that you have  $f(cx)$ . Unlike the  $y$  direction, when  $c > 0$  the graph actually moves the graph closer to the  $y$ -axis and when  $c < 0$ , the graph moves closer to the  $x$ -axis as shown at right.

Compression occurs when  $c > 0$ . Stretching occurs when  $0 < c < 1$ .



$$g(x) = 0.35(x^2)$$

Check for understanding:

How are the following graphs changed from their parent graph?

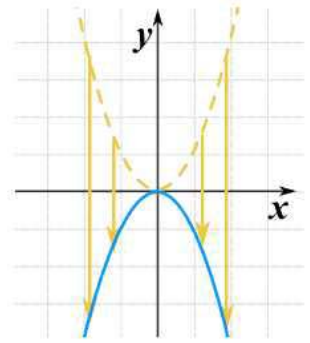
5)  $y = 3x^2$  (parent:  $y = x^2$ )

6)  $y = \frac{1}{2} x^2$  (parent:  $y = x^2$ )

7)  $y = 2(x - 5)^2$  (parent:  $y = x^2$ )

8)  $y = 1/3 x^2 + 4$  (parent:  $y = x^2$ )

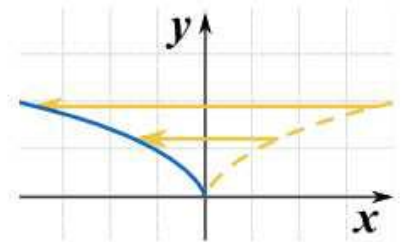
The entire function can be flipped over the x-axis by multiplying the function by -1, as shown below. This is also known as **reflecting** the function over the x-axis. Did you know you can also reflect the function and stretch the function at the same time?



$g(x) = -(x^2)$

This can be accomplished by multiplying the entire function by a number such as -2. The -2 will flip the function over the x-axis and will stretch the graph in the y direction. Pretty cool, huh?

Well - if you can reflect the function over the x-axis, you can do the same over the y-axis by multiplying the x-value by -1. It's hard to tell in the function  $f(x) = x^2$ , so let's use another function  $g(x) = \sqrt{x}$



$g(x) = \sqrt{-x}$

To summarize all of these transformation, look at the table below:

$y = f(x) + C$	<ul style="list-style-type: none"> <li>• <math>C &gt; 0</math> moves it up</li> <li>• <math>C &lt; 0</math> moves it down</li> </ul>
$y = f(x + C)$	<ul style="list-style-type: none"> <li>• <math>C &gt; 0</math> moves it left</li> <li>• <math>C &lt; 0</math> moves it right</li> </ul>
$y = C \cdot f(x)$	<ul style="list-style-type: none"> <li>• <math>C &gt; 1</math> stretches it in the y-direction</li> <li>• <math>0 &lt; C &lt; 1</math> compresses it</li> </ul>
$y = f(Cx)$	<ul style="list-style-type: none"> <li>• <math>C &gt; 1</math> compresses it in the x-direction</li> <li>• <math>0 &lt; C &lt; 1</math> stretches it</li> </ul>
$y = -f(x)$	<ul style="list-style-type: none"> <li>• Reflects it about x-axis</li> </ul>
$y = f(-x)$	<ul style="list-style-type: none"> <li>• Reflects it about y-axis</li> </ul>

Check for understanding:

How are the following graphs translated from their parent graph?

9)  $y = -(x + 4)^2$  (parent:  $y = x^2$ )

10)  $y = -\sqrt{x} + 2$  (parent:  $y = \sqrt{x}$ )

11)  $y = \sqrt{-x} - 3$  (parent:  $y = \sqrt{x}$ )

12)  $y = -2\sqrt{x+4}$  (parent:  $y = \sqrt{x}$ )