Common Core Math II Name Date

Exponential Functions Review

Simplify each expression, and write your final answer with rational exponents.

1) $\sqrt{36s^2} \cdot (s^6)^{1/3}$ 2) $2k^{2/3} \cdot \frac{1}{4}k^{5/6}$ 3) $x\sqrt[4]{16} \cdot 2^4 x$

$6s^3$ $\frac{1}{2}k^{11/6}$ $2^5x^2 = 32x^2$

Simplify each expression, and write your final answer in simplest radical form.

4) $m^{1/2} \cdot m^{4/3}$ 5) $(12n^2 \cdot 24n^{1/4})^3$ 6) $\sqrt[4]{256x^8} \cdot \sqrt{8x^3}$

$m\sqrt[6]{m^5}$ $23887872n^6\sqrt[4]{n^3}$ $8x^3\sqrt{2x}$

* 7) Explain why $16^{13/4} = 16^3 \cdot \sqrt[4]{16}$ is a true statement.

$16^3 \cdot \sqrt[4]{16}$ is the same as $16^3 \cdot 16^{1/4}$. Since $a^m a^n = a^{m+n}$ and $3 + \frac{1}{4} = \frac{13}{4}$, $16^{13/4} = 16^3 \cdot \sqrt[4]{16}$.

8) Fill in the blank to make each statement true.

a. $2x^6 \cdot \underline{4x^{-5}} = 8x$ b. $(10x^{-3})/\underline{2x^{-4}} = 5x$ c. $(\underline{2^{5/2}x^2})^2 = 32x^4$ d. $(\underline{2^{-1}m^{-3}})^{-3} = 8m^9$

9) Write each expression in simplified radical form.

a. $\sqrt[5]{8m^2n^4} \cdot \sqrt[5]{20m^4n}$ c. $\sqrt[3]{k} \cdot k^{6/4}$ e. $-3\sqrt[4]{16y^9}$

$2mn\sqrt[5]{5m}$ $k\sqrt[6]{k^5}$ $-6y^2\sqrt[4]{y}$

b. $\sqrt{72} - \sqrt{75} + \sqrt{98}$ d. $\sqrt{81x^3y^6}$ f. $\sqrt[3]{(b-5)^2(b-5)^4}$

$13\sqrt{2} - 5\sqrt{3}$ $9xy^3\sqrt{x}$ $(b-5)^2$

10) Explain how to calculate the value of $81^{3/4}$ without using a calculator.

$81^{3/4} = (\sqrt[4]{81})^3 = (3)^3 = 27$

11) Find the solution(s) for each of the following equations.

a. $2x^{\frac{4}{3}} - 2 = 160$ d. $\sqrt{2x+1} = -5$ g. $(2x+1)^{1/3} = 1$

no solution 0

$-27, 27$

b. $4x^{1/2} - 5 = 27$ e. $x^{1/6} - 2 = 0$ h. $x^{1/4} + 3 = 0$

64 64 no solution

c. $\sqrt{x+1} = x + 1$ f. $\sqrt{x+2} = x - 18$ i. $\sqrt[3]{2x-4} = -2$

0, -1 23 -2

12) The function $y = 187900\,(1.025)^x$ represents the value of a home x years after purchase. Find the monthly and quarterly rate of appreciation of the home.

$(1.025)^x = (1.025^{\frac{1}{4}})^{4x} = 1.00619$ or 0.619% growth quarterly

$(1.025)^x = (1.025^{\frac{1}{12}})^{12x} = 1.00206$ or 0.206% growth monthly

13) The function $y = 290{,}000\,(0.92)^x$ represents the value of an old home that has been abandoned by its owners x years ago. Find the decay rate of the old home.

$1 - 0.92 = 0.08$

decay rate is 0.08 percent of decay is 8% each year.

14) Use the rules of exponents to find the value of x in each equation.

a. $(2^{x+1})^5 = 2^x$

$5x+5 = x$

$5 = -4x$

$x = -5/4$

b. $(3^{2x})(3^{16}) = 3^{48}$

$2x+16 = 48$

$2x = 32$

$x = 16$

c. $(5^{50})/(5^{40}) = 5^{x-5}$

$50-40 = x-5$

$10 = x-5$

$x = 15$

15) Use your calculator to find the following logarithms.

a. log -100 — no solution

b. log 426 — 2.63

c. log 100 — 2.

d. log (0.0001) — -4

e. log 3.45 — 0.54

16) Use your knowledge of exponents and logarithms to solve these equations two ways.

a. $3(10^x) = 3{,}000$

$10^x = 10^3$

$\log 1000 = x$

$x = 3$

b. $10^{2x-1} = 100$

$10^{2x-1} = 10^2$

$\log 100 = 2x-1$

$x = 1.5$

c. $10^{2x} - 3 = 997$

$10^{2x} = 10^3$

$\log 1000 = 2x$

$x = 1.5$

d. $10^x = 1$

$10^x = 10^0$

$\log 1 = x$

$x = 0$

e. $-2(10)^{x+4} = -.002$

$10^{x+4} = 10^{-3}$

$\log .001 = x+4$

$x = -7$

f. $10^{x/2} = 25$

$10^{x/2} = 10^{1.398}$

$\log 25 = \frac{x}{2}$

$x = 2.796$

g. $\frac{3}{2}(10)^{x+2} = 1500$

$10^{x+2} = 10^3$

$\log 1000 = x+2$

$x = 1$

h. $3(10)^{x+4} + 3 = 15$

$10^{x+4} = 10^{.602}$

$\log 4 = x+4$

$x = -3.398$

17) A 100 milligram sample of Carbon-10 has a half-life of 19.29 seconds. Write an exponential function to model its decay. Let x= time in minutes and f(x) = the amount of Carbon-10 remaining in the sample.

$y = 100\,(0.5)^{3.11x}$

Since x is time in minutes, you must multiply by 60 to get time in seconds & then divide by 19.29.

18) Create a real world scenario that could be modeled by the function $f(x) = 40000 \cdot 0.95^x$. In your scenario, make sure to address percentage of exponential growth or decay and initial value.

The initial value of a car is \$40,000, and it is depreciating 5% each year. Write an exponential model in which x is time in years and f(x is the value of the car.

19) A popular antique is gaining value because it is so hard to find. In 1985 its value was $125, and in 2000 its value was $1925.90.

a. Find an explicit exponential function to model the information – show your work.

$1925.90 = 125\, b^{(2000-1985)}$ → $15.4072 = b^{15}$

$1925.90 = 125\, b^{15}$ — $b = 1.2$

$y = 125(1.2)^x$

x = years since 1985

b. Write a recursive (NOW-NEXT) function to model the data.

NEXT = NOW (1.2), start at 125

c. Determine the percentage of yearly appreciation.

20%

d. If the same trend continues, how much was the antique worth in 2010?

$11924.50

Use what you know about solving exponential equations with base 10 to solve the following growth problem.

20) In a drop of pond water, there are 18 protozoa. Ten hours later, there are 180 protozoa in the dish. *P(t)* = $18(10^{0.1t})$ provides an exponential growth model that matches these data points.

a. Verify that the model P(t) = $18(10^{0.1x})$ represents the information provided.

$P(0) = 18(10^{0.1 \cdot 0})$ Initial value
$= 18(10^0)$ is 18.
$= 18$

$P(10) = 18(10^{0.1 \cdot 10})$ After 10 hours
$= 18(10^1)$ there are 180
$= 180$ protozoa.

b. Use the given function to estimate the time when the bacteria population would be expected to reach 500,000. 44.4 hours later

i. Explain how to find the time by numerical or graph estimation.

Let $y_1 = 18(10^{0.1t})$ & $y_2 = 500000$. Graph and find the point of intersection.

ii. Explain how to find the time by using common logarithms and algebraic reasoning.

$500000 = 18(10^{0.1t})$ Divide both sides by 18 and then take the log of both sides. Finally, divide each side by 0.1.

c. What is the theoretical domain of the function?

all real numbers

d. What is the practical domain of the function?

all x-values greater than or equal to 0

e. What is the range of the function?

The theoretical range is all $y > 0$, practically only whole numbers greater than 18.

f. What are the intercepts of the function and what do they mean in the context of the problem?

There is no x-intercept, but the y-intercept is 18 because of the initial amount of protozoa.

g. What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem?

The function increases on the entire domain because the population of protozoa is growing.

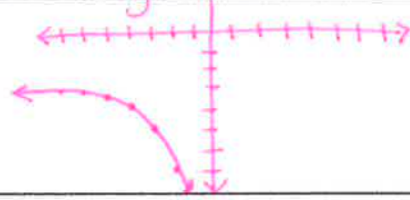
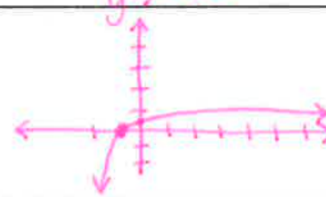
21) For the function $f(x) = (0.75)^x - 1$ evaluate the following:

a. $f(-1) =$ $\frac{1}{3}$ b. $f(0) =$ -1 c. $f(5) =$ $-\frac{781}{1024}$ ≈ -0.7627 d. $f(2) =$ $-\frac{7}{16}$ $= -0.4375$

22)

Describe the effect on the graph when…	$y = a \cdot b^{x+c} + d$	$y = a \cdot \log(bx + c) + d$
a is negative	as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$, $y \to 0$	as $x \to \infty$, $y \to -\infty$ as $x \to -\infty$, $y \to \infty$
a increases	The y-intercept increases and the graph is steeper.	The function grows more quickly or is more steep. Higher y-intercept
b increases	The y-intercept stays the same, but the graph is steeper.	The y-intercept stays the same but the graph is a little steeper.
c decreases	The graph moves right	The graph moves right.
d increases	The graph moves up	The graph moves up.

23) For each of the functions describe the key characteristics.

	$y = -2^{x+4} - 3$	$y = \log(x+2)$
Domain	all real numbers	$x > -2$
Range	$y < -3$	all real numbers
Asymptotes (if any)	$y = -3$	$x = -2$
Zeros (if any)	none	$(-1, 0)$
End behavior as $x \to \infty$	$y \to -\infty$	$y \to \infty$
End behavior as $x \to -\infty$	$y \to 3$	$y \to -\infty$
Sketch of the function		

25) Given the function $y = \frac{4}{x-3}$, answer the following questions.

a. What is the inverse of the function?

$y = \frac{4}{x} + 3$

$y = \frac{4}{\frac{4}{x} + 3 - 3}$

b. How can you verify algebraically that the functions are inverses?

Substitute $\frac{4}{x} + 3$ for x in the initial function and simplify. If your final result is $y = x$, they're inverses.

$y = \frac{4}{\frac{4}{x}}$

c. How can you verify graphically that the functions are inverses?

Graph both to show each is a reflection of the other over the line $y = x$.

$y = 4 \cdot \frac{x}{4}$

$y = x$

26) The following table gives some ordered pairs generated using the function g(x). Create a table containing points from $g^{-1}(x)$.

x	g(x)
4	-7
1	-4
0	3
-2	12

x	$g^{-1}(x)$
-7	4
-4	1
3	0
12	-2