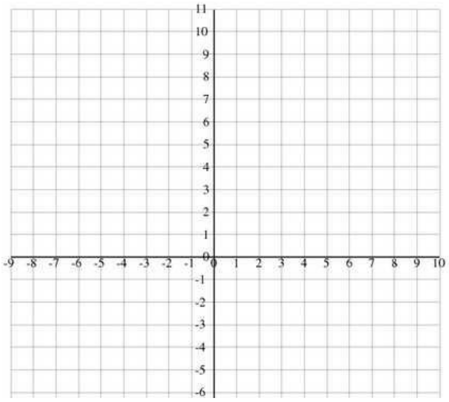
Day 1: Counting Methods, Permutations & Combinations

Warm-Up:

1. Given the equation $y = 4 + \sqrt{x+2}$ draw the graph, being sure to indicate at least 3 points clearly.



Then determine the following:

a. Identify its vertex ___________

b. Identify its domain ________________

c. Identify its range ________________

d. How is this function translated from its parent graph?

____

e. If this graph was translated to the right 5 units, what would the new equation be? _____________________

Solve

2. $2\sqrt[3]{(x-1)^4} + 4 = 36$

3. $\sqrt{x+7} - x = 1$

Notes Day 1: Counting Methods, Permutations & Combinations

I. <u>Introduction</u>

Probability Defined:

II. <u>Basic Counting Methods for Determining the Number of Possible Outcomes</u>

A. Fundamental Counting Principle:

a. Tree Diagrams:

<u>Example #1:</u> LG will manufacture 5 different cellular phones: Ally, Extravert, Intuition, Cosmos and Optimus. Each phone comes in two different colors: Black or Red. Make a tree diagram representing the different products. How many different products can the company display?

b. In general: If there are ___ ways to make a first selection and ___ ways to make a second selection, then there are ___________ ways to make the two selections simultaneously. This is called the Fundamental Counting Principle.

<u>Ex #1 above:</u> 5 different types of phones in 2 different colors.
How many different products will LG display?

Ex #2: Elizabeth is going to completely refurbish her car. She can choose from 4 exterior colors: white, red, blue and black. She can choose from two interior colors: black and tan. She can choose from two sets of rims: chrome and alloy. How many different ways can Elizabeth remake her car? Make a tree diagram and use the Counting Principle.

Ex #3: Passwords for employees at a company in Raleigh NC are 8 digits long and must be numerical (numbers only). How many passwords are possible? (Passwords cannot begin with 0)

B. Permutations

a. Two characteristics: 1. Order ___________

2. No item is used _______ _______ ________

Example #1: There are six "permutations", or arrangements, of the numbers 1, 2 and 3. What are they?

Example #2: How many ways can 10 cars park in 6 spaces? The other four will have to wait for parking spot. ☺ (Use the Fundamental Counting Principle)

b. Formula: If we have a large number of items to choose from, the fundamental counting principle would be inefficient. Therefore, a formula would be useful.

First we need to look at **"factorials"**.

Notation: _____ stands for n factorial.

Definition of n factorial:
For any integer n>0, n! = ______________

If n=0, 0! = ____.

Example #2 (revisited): We could rewrite the computation in our example as follows:

$$10\bullet9\bullet8\bullet7\bullet6\bullet5 = \frac{10\bullet9\bullet8\bullet7\bullet6\bullet5\bullet4\bullet3\bullet2\bullet1}{4\bullet3\bullet2\bullet1} = \_\_\_\_\_\_\_\_\_\_$$

Furthermore, notice that $\frac{10!}{4!}$ = _________

So, the number of permutations (or _________________________) of 10 cars taken 6 at a time is _____.

Generally, the ***Number of Permutations*** of n items taken r at a time, $_nP_r$ =

- How to do on the calculator:

c. Example #3 In a scrabble game, Jane picked the letters A, D, F, V, E and I. How many permutations (or ____________________) of 4 letters are possible?

Practice Problems:

1. Evaluate: (By hand, then using $_nP_r$ function on the calculator to check your answer)
   a. $_{10}P_3$ b. $_9P_5$

2. How many ways can runners in the 100 meter dash finish 1st (Gold Medal), 2nd (Silver) and 3rd (Bronze Medal) from 8 runners in the final? **NOTE:** This is a permutation because the people are finishing in a position.

C. Combinations

a. Two characteristics: 1. Order _________ _________ matter

2. No item is used ________ ________ _________

Example: While creating a playlist on your Ipod you can choose 4 songs from an album of 6 songs. If you can choose a given song only once, how many different combinations are possible? (List all the possibilities)

b. Formula:

Making a list to determine the number of combinations can be time consuming. Like permutations, there is a general formula for finding the number of possible combinations.

The ***Number of Combinations*** of n items taken r at a time,
$_nC_r$ =

- How to on the calculator:

Practice Problems:

1. Evaluate: (by hand then using ${}_nC_r$ function on the calculator to check your answer)

   a. ${}_4C_2$ b. ${}_7C_3$ c. ${}_8C_8$

2. A local restaurant is offering a 3 item lunch special. If you can **choose 3 or fewer items** from a total of 7 choices, how many possible combinations can you select?

3. A hockey team consists of ten offensive players, seven defensive players, and three goaltenders. In how many ways can the coach select a starting line up of three offensive players, two defensive players, and one goaltender?

<u>Mixed Practice:</u> Indicate if the situation is a permutation or a combination. Then, solve.

a. In a bingo game 30 people are playing for charity. There are prizes for 1st through 4th. How many ways can we award the prize? **Permutation or Combination (circle 1)**

b. From a 30-person club, in how many ways can a President, Treasurer and Secretary be chosen? **Permutation or Combination**

c. In a bingo game 30 people are playing for charity. There are two $50 prizes. In how many ways can prizes be awarded? **Permutation or Combination**

d. How many 3-digit passwords can be formed with the numbers 1, 2,3,4,5 and 6 if no repetition is allowed? **Permutation or Combination**

e. Converse is offering a limited edition of shoes. They are individually made for you and you choose 4 different colors from a total of 25 colors. How many shoes are possible? **Permutation or Combination**

f. A fast food chain is offering a $5 box special. You can choose no more than 5 items from a list of 8 items on a special menu. In how many ways could you fill the box? **Permutation or Combination**

Day 2: Basic Probability

Warm-Up:

1. Suppose you are asked to list, in order of preference, the three best movies you have seen. If you saw 20 movies, in how many ways can the 3 best be chosen and ranked?

2. There are 6 women and 5 men interviewing for 4 cashier positions at Walmart.

   a) In how many ways can the 4 positions be filled?

   b) In how many ways can the positions be filled if all women are hired?

   c) In how many ways can the positions be filled if 2 women and 2 men are hired?

3. How many distinguishable permutations are possible using the letters of the following words:

   a) ATHENS c) SUBSTITUTE

   b) BASKETBALL d) ICICLE

Notes Day 2: Basic Probability

Sample Space:____
____

List the sample space, S, for each of the following:

a. Tossing a coin	b. Rolling a six sided die	c. Drawing a marble from a bag containing two red, three blue, and one white marble

Intersection of two sets ($A \cap B$):____
____

Union of two sets ($A \cup B$): ____
____

Example: Given the following sets, find A ∩ B and A ∪ B.

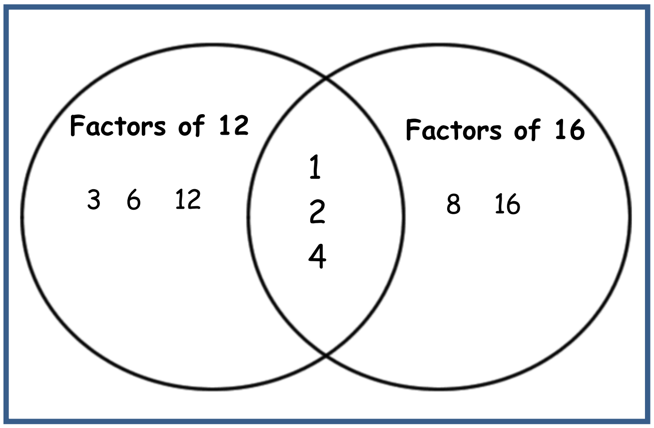
A = {1,3,5,7,9,11,13,15} B = {0,3,6,9,12,15}

A ∩ B = ______________________ A ∪ B = _______________________

Venn Diagram: ____ Picture:

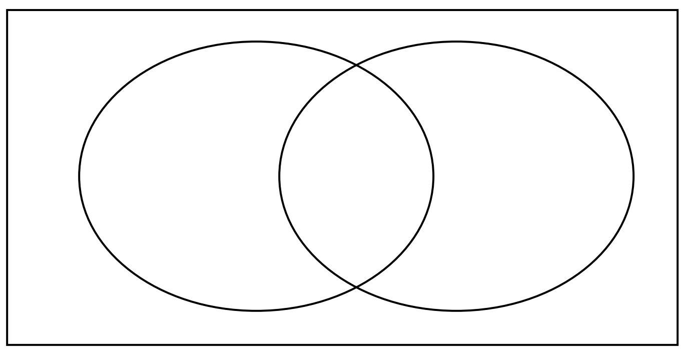
____

Example: Use the Venn Diagram to answer the following questions:



1. What are the elements of set A?
2. What are the elements of set B?
3. Why are 1, 2, and 4 in both sets?
4. What is A ∩ B?
5. What is A ∪ B?

Example: In a class of 60 students, 21 sign up for chorus, 29 sign up for band, and 5 take both. 15 students in the class are not enrolled in either band or chorus.



6. Put this information into a Venn Diagram. If the sample space, S, is the set of all students in the class, let students in chorus be set A and students in band be set B.
7. What is A ∪ B? __________________
8. What is A ∩ B? __________________

Compliment of a set: ____

____

- Ex: S = {…-3,-2,-1,0,1,2,3,4,…} A = {…-2,0,2,4,…}
- If A is a subset of S, what is A^C? ___________________________

Example: Use the Venn Diagram from the Band & Chorus problem above to find the following:

9. What is A^C? ________________ B^C? _______________

10. What is $(A \cap B)^C$? ________________________

11. What is $(A \cup B)^C$? ________________________

Basic Probability
Probability of an Event: P(E) = __________________________________

Example 1: A spinner has 4 equal sectors colored yellow, blue, green and red. After spinning the spinner, what is the probability of landing on each color?

P(yellow) = P(green) =

P(blue) = P(red) =

<u>You Try!</u>

Example 2: A single 6-sided die is rolled. What is the probability of each outcome?
What is the probability of rolling an even number? Of rolling an odd number?

P(1) = P(2) = P(3) = P(4) = P(5) = P(6) =

P(even)= P(odd) =

Probability of A^c
Note that **$P(A^C)$** is every outcome **except (or not)** *A*, so we can find $P(A^C)$ by finding ____________________.

Why do you think this works?____

Example 3: A pair of dice is rolled. What is the probability of **NOT** rolling doubles?

*There are _____ ways to roll doubles.

P(doubles) =

P(NOT doubles) =

Example 4: A pair of dice are rolled. What is the probability of rolling a sum of 10 or less?
*What is the complement of rolling "10 or less"?

Example 5: An experiment consists of tossing three coins.

12. List the sample space for the outcomes of the experiment.

13. Find the following probabilities:

a. P(all heads) ___________________
b. P(two tails) ___________________
c. P(no heads) ___________________
d. P(at least one tail) ______________
e. How could you use compliments to find d?

____

____

You Try!

Example 6: A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

14. List the sample space for this experiment.

15. Find the following probabilities:

a. P(red) _____________
b. P(blue or white) _____________
c. P(not yellow) _____________

Note that we could either count all the outcomes that are not yellow or we could think of this as being 1 – P(yellow).

Why is this? ____

Example 7: A card is drawn at random from a standard deck of cards. Find each of the following:

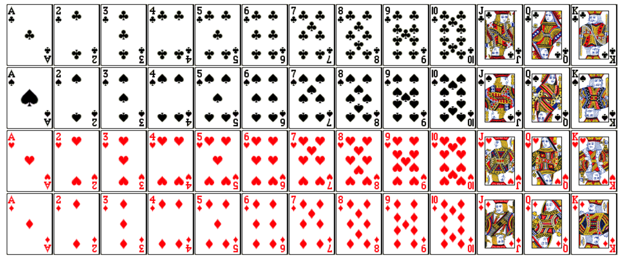
16. P(heart) ______________

17. P(black card) ______________

18. P(2 or jack) ______________

19. P(not a heart) ______________

Black
Black
Red
Red



Notes

Odds: The **odds** of an event occurring are equal to the ratio of ______________ to ____________.
Odds = ____

20. The weather forecast for Saturday says there is a 75% chance of rain. What are the odds that it will rain on Saturday?

- What does the 75% in this problem mean?

The favorable outcome in this problem is that it rains.

- Odds(rain) =

You Try!

21. What are the odds of drawing an ace at random from a standard deck of cards?

22. A gumball machine contains gumballs of five different colors: 36 red, 44 white, 15 blue, 20 green, and 5 orange. The machine dispenser randomly selects one gumball. What is the probability that the gumball selected is:

    a) Green?

    b) Not green?

    c) Not orange?

    d) Orange?

    e) Not a color in the flag of the USA?

    f) Red, white or blue?

Day 3: Independent and Dependent Events

Warm-up:

1. If you have a standard deck of cards in how many different hands exist of:
   5 cards? (Show work by hand) 2 cards? (Show work by hand)

How many ways are there to…

2. Pick a team of 3 people from a group of 10.
3. Choose 3 desserts from a menu of 8 desserts.
4. Choose a winner and a runner up from the 40 Miss Pickle Princess contestants.
5. Arrange the letters of the word FACTOR
6. Choose two jelly beans from a bag of 15?
7. Assign the part of a play to the 4 lead characters from a group of 30 who tried out.

Day 3: Independent and Dependent Events

Remember, the probability that an event E will occur is abbreviated _____________.

Review:

Ex 1: If you are dealt one card from a standard 52-card deck, find the probability of getting a king.

Ex 2: Find the probability of rolling a number greater than 2 when you roll a die once.

Vocabulary and Example:

There are 7 marbles in a bag. You draw 2 marbles from the bag using the following scenarios:

Experiment 1: Draw one marble. Put it back. Draw a marble again.

Draw 2 _________ affected by draw 1.

When the outcome of a second event **is not** affected by the outcome of the first event, then the two events are called ______________________ events.

Experiment 2: Draw one marble. Then draw another without replacing the first.

Draw 2 _________ affected by draw 1.

When the outcome of a second event **is** affected by the outcome of the first event, then the two events are called ______________________ events.

If A and B are independent events, then

P(A and B) = _______________

Example 1: Suppose 5 marbles in the bag are yellow and 2 marbles are green. You draw 1 marble and put it back before drawing a second. Find the probability that the marble color in both draws is yellow.

P(yellow and yellow) =

If A and B are dependent events, then

P(A, then B) = _______________

Example 2: Suppose 5 marbles in the bag are yellow and 2 marbles are green. You draw 1 marble and then another without putting back the first marble. Find the probability that both marbles are yellow.

P(yellow, then yellow) =

Practice: Classifying Events as Dependent or Independent

1) Pick a cookie from the party platter. Then pick another cookie from the same platter.

2) A number from 1 to 31 is selected at random. Then a month is selected at random.

3) A grade level from K-12 is selected at random. Then one of the remaining grade levels is selected at random.

4) Select a bag of chips at random from the pile. Change your mind and return it. Then pick another bag of chips at random.

Practice: Calculating Probabilities of Independent and Dependent Events

A game board in your closet has 7 purple game pieces, 4 red game pieces, and 3 green game pieces. You randomly choose one game piece and then replace it. Then you choose a second game piece. Find each probability.

1) P(red and green)

2) P(green and purple)

3) P(both red)

You are folding the socks from the laundry basket, which contains 6 brown socks, 2 blue socks, and 5 black socks. You pick one sock at a time and don't replace it. Find each probability.

4) P(blue, then black)

5) P(brown, then blue)

6) P(both black)

Tree Diagrams

A ____________________ __________ is an event that is the result of more than one outcome.

Ex 1:

To determine probabilities of compound events, we can use ________ _________________.

For tree diagrams, we place probabilities on the branches and words after the branches. Like the counting principle, you multiply to find the overall probability.

Ex 2: Create a Tree Diagram for the following scenario:

There is a 60% chance of rain on Wednesday. If it rains, the track team has a 70% chance of winning. If it doesn't rain, there is a 95% chance that the track team will win.

To calculate probabilities using tree diagrams, we use the ______________________

__________ **for Compound Events.**

To use this rule, all of the separate outcomes that make up the compound event must be ___________________ ___________, which means that the probabilities of the outcomes do not affect one another.

To calculate probability of compound events:

1st)

2nd)

To fill in the probabilities for the branches not stated in our problem, we'll need to use the "______________________" of each probability.

Ex 3: In the table below, fill in the remaining outcomes for Example 2 above. Then, calculate the probabilities for each outcome.

Outcome	Calculations	Probability
Rain and Track Team Wins	(.60)(.70) = .42	42%

Ex 4: a) What is the probability that the track team wins?

b) What is the probability the team wins *given* that it rains = P(win | rain)?

Practice

Ex 5: A student in Buffalo, New York, made the following observations:

- Of all snowfalls, 5% are heavy (at least 6 inches).
- After a heavy snowfall, schools are closed 67% of the time.
- After a light (less than 6 inches) snowfall, schools are closed 3% of the time.

a) Create a tree diagram for the scenario, listing all possibilities and probabilities.

b) Find the probability that snowfall is light and schools are open.

c) Find the probability that there is snowfall and schools are open.

d) Find P(schools open, given heavy snow) = P(schools open | heavy snow).

Practice of "AND" Probability with Tree Diagrams

You have a special deck of 20 cards for a game. This deck contains cards with equal amounts of the following shapes: heart, circle, square, and triangle

Part A: You choose a card, put it back, and then choose another card.
Calculate the following and express probabilities as %, rounding to the hundredths place.

1) Create a tree diagram to model the different possibilities and probabilities for calculating the probability of choosing a heart, then a circle.

2) P(heart then circle)

3) P(not heart, then circle)

4) P(circle on 2^{nd} draw)

5) P(circle |heart)

6) P(not heart , not circle)

Part B: Suppose now you play with these cards again, but you choose the second card without replacement.

1) Create a tree diagram to model the probabilities and possibilities of choosing a heart then a circle.
Hint: be careful with your probabilities (reread Part B)

2) Why does part of your tree have different probabilities than it did in Part A?

3) P(heart then circle)

4) P(not heart then circle)

5) P(circle on second draw)

6) P(circle | heart)

7) P(not heart, not circle)

8) Why are your answers in this part different from your answers in part A?

Day 4: Quiz Day

Warm-Up:

1. Your I-tunes card has enough for 3 of the 7 songs you want. In how many ways could you pick the songs?

2. We use 10 digits in our number system. How many 4-digit "numbers" can be formed if no digits are repeated ? (Zero is allowed in any position)

3. Confirm your answer to #2 using the Fundamental Counting Principle.

4. Bad Frog Yogurt lets you pick 4 or fewer toppings from 40 choices and save 50 cents off of your order. How many ways can you get the savings?

5. Create a tree diagram to show the sample space for flipping a coin four *times.*

6. Using your answer to #5, what is the probability that all four "flips" are heads?

Notes Day 5: Probability of Mutually Inclusive and Exclusive Events

Probability of an event NOT occurring

The probability that an event E will not occur is equal to one __________ the probability that it will occur.

P(not E) = ___________________________

Ex 1: Find the probability that you choose a number from 1 to ten that is not 6.

Ex 3: You draw a card that is not a red face card (Jack, Queen, King)

Ex 2: Find the probability that you deal a card that is not a diamond.

Ex 4: You select someone in the class who is not wearing jeans.

Ex 5: In the classic lottery game, each player chooses 6 different numbers from 1 to 48. If all of the numbers match the 6 picked, they win. What is the probability of not winning?

"OR" Probability Discovery

Part A: Fifteen plastic squares are placed in a box: four blue, seven red, and four brown. If one plastic square is chosen at random from the box, calculate the following:

Blue	Blue	Red	Red	Brown
Blue	Red	Red	Red	Brown
Blue	Red	Red	Brown	Brown

1) P(Blue) 2) P(Red) 3) P(Blue or Red)

4) P(Blue) 5) P(Brown) 6) P(Blue or Brown)

7) P(Brown) 8) P(Red) 9) P(Brown or Red)

10) How could you use the answers in the first two columns to get the answer to the third column? (Example: how could you use #1 and #2 to get #3?)

11) Suppose you need to write a formula for P(A or B) using P(A), P(B), mathematical symbols, and the information from the problems above. What could the formula be?

Definitions:

Mutually Exclusive: Two or more events that cannot occur at the same time.
Example a: the scenario above involved items that are mutually exclusive
Example b: a number on a die being even and odd

Mutually Inclusive: Two events that can occur at the same time.
Example a: a number on a die being even and less than five

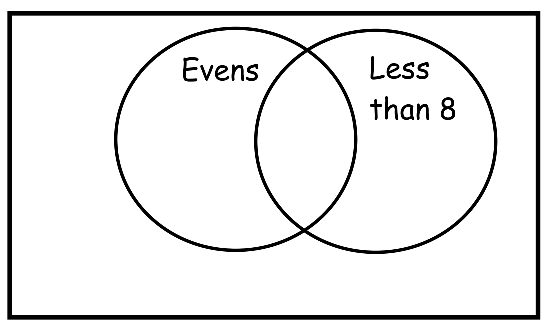
The probability of mutually exclusive scenarios require more thought and careful attention.

Part B: Suppose eleven plastic squares are placed in a box with the numbers 1 – 11 on them. Use this information to calculate the following:

1	2	3	4	5	6
7	8	9	10	11	

1) P(Even) 2) P(Less than 8)

3) Create a Venn Diagram for Evens and Less than 8



4) P(Even and Less than 8)

5) Even U Less than 8 (remember U means "or" so this means numbers that are even, less than 8, or both)

6) P(Even U Less than 8)

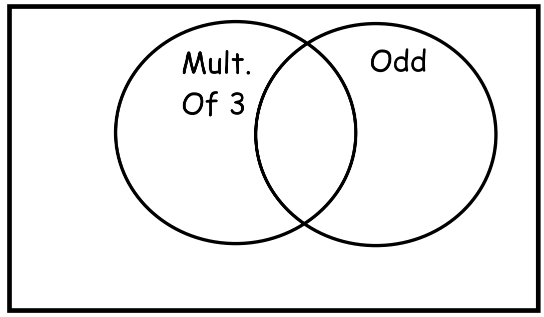
7) Why does it not make sense to simply add P(Even) and P(Less than 8) to get P(Even U Less than 8) ?

8) How could you have used your answers from P(Even), P(Less than 8) and P(Even and Less than 8) to get P(Even U Less than 8)?

1	2	3	4	5	6
7	8	9	10	11	

9) P(Multiple of 3) 10) P(Odd)

11) Create a Venn Diagram for Multiple of 3 and Odd



12) P(Multiple of 3 $\cap$ Odd)
(Hint: remember $\cap$ means "and")

13) Multiple of 3 U Odd

14) P(Multiple of 3 U Odd)

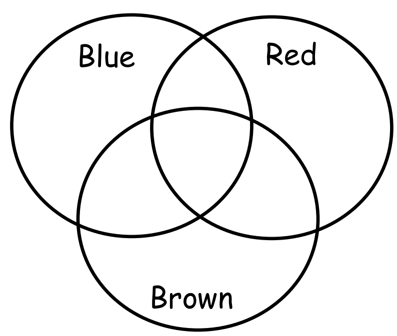
15) How could you have used your answers from P(Multiple of 3), P(Odd) and P(Multiple of 3 $\cap$ Odd) to get P(Multiple of 3 U Odd)?

16) Suppose you need to write a NEW formula for P(A or B), now that you see that your prior formula from Part A will not always work! Using P(A), P(B), P(A $\cap$ B), mathematical symbols, and the information in the problems, what could be the formula for P(A U B)?

Part C:
Let's take a look back at the scenario from Part A to see if our new formula and methods will work for that scenario too.

Blue	Blue	Red	Red	Brown
Blue	Red	Red	Red	Brown
Blue	Red	Red	Brown	Brown

1) Create a Venn Diagram for blue and red and brown.



2) P(Blue $\cap$ Red) 3) P(Blue $\cap$ Brown)

4) P(Brown $\cap$ Red)

Use the formula you found at the end of Part B and the work from the past few questions to calculate the following:

5) P(Blue U Red) 6) P(Blue U Brown) 7) P(Brown U Red)

8) Compare the last there answers to your answers for the "OR" probabilities in Part A. They should be the same. If they are not, check your work.

Summary:

Mutually Exclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or you roll a 2?

- Can these both occur at the same time? Why or why not? ________________________________

  ____

Mutually Exclusive Events: ____

+ The probability of two mutually exclusive events occurring at the same time, P(A and B), is ____

To find the probability of one of two **<u>mutually exclusive</u>** events occurring, use the following formula:

____

Mutually Inclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an odd number or a number less than 4?

- Can these both occur at the same time? If so, when? ____________________________________

  ____

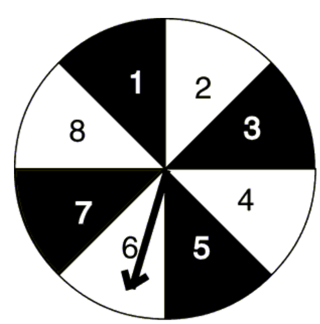
Mutually Inclusive Events: ____

Probability of the Union of Two Events: The Addition Rule: ______________________________

____

____

Ex/ Are the events mutually exclusive or mutually inclusive? Explain.



1) Spinning a 4 or a 6 at the same time on a single spin.

2) Spinning an even number or a multiple of 3 at the same time on a single spin.

3) Spinning an even number or a prime number on a single spin.

4) Spinning an even number or a number less than 2 on a single spin.

Examples of "OR" probability:

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?

2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

3. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the first 10 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

4. A bag contains 26 tiles with a letter on each, one tile for each letter of the alphabet. What is the probability of reaching into the bag and randomly choosing a tile with one of the last 5 letters of the alphabet on it or randomly choosing a tile with a vowel on it?

EX. Spinning a 4 or 6 on a 1-8 spinner (numbers 1, 2, 3, 4, 5, 6, 7, and 8 occur on the spinner). Are they mutually exclusive or mutually inclusive?

Practice/Examples:

1. If you randomly chose one of the integers 1 – 10, what is the probability of choosing either an odd number or an even number?
   Are these mutually exclusive or mutually inclusive events? Why?

   Complete the following statement: P(odd or even) = P(______) + P(______)

   Now fill in with numbers: P(odd or even) = ________ + _________ = ___________________

   Does this answer make sense?

2. Two fair dice are rolled. What is the probability of getting a sum less than 7 or a sum equal to 10?
   Are these events mutually exclusive or mutually inclusive? Why?

Sometimes using a table of outcomes is useful. Complete the following table using the sums of two dice:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4				
3	4					
4						
5						
6						

P(getting a sum less than 7 OR sum of 10) = ____

This means ____

Day 6: Tree Diagrams, Conditional Probability, and Two Way Tables

Warm-Up:

Use the fundamental counting principle, permutation or combination formulas to answer the following.

1) As I'm choosing a stocking for my nephew, I'm given a choice of 3 colors for the stocking itself. I plan to have his name embroidered. I have 8 thread choices and 5 font choices. How many variations of stocking can be made for my nephew?

2) I have a candy jar filled with 250 different candies. How many ways can I grab a handful of 7 yummy confections to eat?

3) The VonTrapp family is taking pictures (they have 7 children). How many ways can we line the children up?

4) The VonTrapp children are being ornery. Kurt and Brigitta have to be on either side of the picture (away from one another). Now, how many ways can I line the children up?

5) I need to choose a password for my computer. It must be 5 characters long. I can choose any of the 26 letters of the alphabet (lowercase only) or any number as a character. How many possible passwords can I create?

Notes Day 6: Two Way Frequency Tables and Conditional Probability

Conditional Probability:

- Contains a ________________ that may _________ the sample space for an event
- Can be written with the notation __________, which is read "the probability of event B, __________ event A"
- How likely is one event to happen, given that another event _______ happened?
- Percentages/probability based on the ______ or ____________ total of the given event
- More complex "given" problems may require use of this formula:

Examples of Conditional Probability:

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

2. A bag contains 6 blue marbles and 2 brown marbles. One marble is randomly drawn and discarded. Then a second marble is drawn. Find the probability that the second marble is brown given that the first marble drawn was blue.

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

4. **You Try!** In Mrs. Walden's class, 65% of the students have brown hair, 30% have green eyes, and 8% have both brown hair and green eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has green eyes?

Using Two-Way Frequency Tables to Compute Conditional Probabilities

1. Suppose we survey all the students at school and ask them how they get to school and also what grade they are in. The chart below gives the results. Complete the two-way frequency table:

	Bus	Walk	Car	Other	Total
9^{th} or 10^{th}	106	30	70	4	
11^{th} or 12^{th}	41	58	184	7	
Total					

Suppose we randomly select one student.

a) What is the probability that the student walked to school?

b) P(9^{th} or 10^{th} grader)

c) P(rode the bus OR 11^{th} or 12^{th} grader)

d) What is the probability that a student is in 11th or 12th grade *given that* they rode in a car to school?

e) What is P(Walk|9th or 10th grade)?

2. The manager of an ice cream shop is curious as to which customers are buying certain flavors of ice cream. He decides to track whether the customer is an adult or a child and whether they order vanilla ice cream or chocolate ice cream. He finds that of his 224 customers in one week that 146 ordered chocolate. He also finds that 52 of his 93 adult customers ordered vanilla. Build a two-way frequency table that tracks the type of customer and type of ice cream.

	Vanilla	Chocolate	Total
Adult			
Child			
Total			

a) Find P(vanilla)

b) Find P(child)

c) Find P(vanilla|adult)

d) Find P(child|chocolate)

3. A survey asked students which types of music they listen to? Out of 200 students, 75 indicated pop music and 45 indicated country music with 22 of these students indicating they listened to both. Use a Venn diagram to find the following

a) Probability that a randomly selected student listens to pop music given that they listen country music.

b) P(pop U country)

c) P(pop $\cap$ country)

d) P(country ^{c})

Using Conditional Probability to Determine if Events are Independent

If two events are statistically independent of each other, then:

____

Let's revisit some previous examples and decide if the events are independent.

1. You are playing a game of cards where the winner is determined by drawing two cards of the same suit without replacement. What is the probability of drawing clubs on the second draw if the first card drawn is a club?

- Are the two events independent?
- Let drawing the first club be event A and drawing the second club be event B.

2. You are playing a game of cards where the winner is determined by drawing tow cards of the same suit. Each player draws a card, looks at it, then replaces the card randomly in the deck. Then they draw a second card. What is the probability of drawing clubs on the second draw if the first card drawn is a club? Are the two events independent?

3. In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment.

   - If the student has brown hair, what is the probability that the student also has brown eyes?

   - Are event A, having brown hair, and event B, having brown eyes, independent?

4. **You Try!** Using the table from the ice cream shop problem, determine whether age and choice of ice cream are independent events.

"Given" Discovery- Using formula with tree diagrams

Some "given" problems can be solved by looking at the information. Others require this formula:

$$P(A|B)=\frac{P(A\,and\,B)}{P(B)}$$

Ex: Suppose you manage a restaurant that serves chicken wings that are mild or hot, and boneless or regular. From your experience you know that of boneless wings bought, 75% of them are mild, and of the regular wings bought, 70% are hot. Only 4 out of 10 costumers buy boneless wings.

- Use the information to calculate the following. (Hint: in some problems you can use the "given")

1) Create a tree diagram for the scenario displaying all the possibilities and probabilities.

2) P(boneless and hot wings)

3) P(hot | boneless)

4) P(hot)

5) P(boneless |hot)

6) P(mild wings)

7) If a person orders regular wings, what is the probability they choose mild?

8) P(boneless |mild)

9) Of the boneless wings, what is the probability someone orders mild?

*NOTE: None of your prior answers should be equal. If they are, check your work!

Day 7: Probability Review

Warm Up:

1. For a radio show, a DJ can play 4 songs. If there are 8 to select from, in how many ways can the program for this show be arranged?

2. An election ballot asks voters to select no more than three city commissioners but at least one from a group of six candidates. In how many ways can this be done?

3. Consider a set of cards labeled 1-10. Let set A = even numbers and set B = # greater than 8. Find the probability of A or B.

4. Using the situation from problem #3, what is the probability you select an even number given you selected a number greater than 8?

More Review:

1. When rolling a die twice, find the probability of rolling an odd number then a multiple of 2.

2. When rolling a die once, find the probability of rolling an number greater than 3 or a multiple of 3.

3. Mike noticed that a lot of the students taking the ACT were also taking the SAT. In fact, " of the 80 students in his grade, 32 students were taking the ACT, 48 students were taking the SAT, and 12 students took both the ACT and the SAT.

   a) Draw a Venn diagram and help Mike with his calculations.

   b) Calculate P(ACT U SAT)

   c) Calculate P(ACT $\cap$ SAT)

   d) Calculate the probability of selecting a student at random who was either taking the ACT or SAT, but not both.

   e) Calculate P(Not taking any test^c)

Notes Day 8: Quiz Review and Quiz

Warm-Up:

Calculate the following values. For #1 and #2, show all work by hand with the formulas and factorials.

1. The ski club with ten members is to choose three officers captain, co-captain, and secretary. How many ways can those offices be filled?

2. You are on your way to Hawaii and of 15 possible books, you can only take 10. How many different collections of 10 books can you take?

3. Dominoes offers a deal that says you can order a pizza with at most 5 toppings for $9.99. If there are a total of 12 types of toppings, how many different pizzas could you order?

4. On a fair dice, what is the probability of rolling a multiple of 3 or a number greater than 4? Are these events mutually inclusive or exclusive?

5. A family who breeds dogs made the following observations:
   - 30% of their dogs are males
   - 15% of females are brown colored
   - 56% of males are not brown colored

   What is the probability there is a brown male?

More Quiz Review

1) When rolling a die twice, find the probability of rolling an even number then a multiple of 3.

2) When rolling a die once, find the probability of rolling an even number or a multiple of 3.

3) Jack is a student in Bluenose High School. He noticed that a lot of the students in his math class were also in his chemistry class. In fact, of the 60 students in his grade, 28 students were in his math class, 32 students were in his chemistry class, and 15 students were in both his math class and his chemistry class.

   a) Draw a Venn diagram and help Jack with his calculations.

   b) Calculate P(math U chemistry)

   c) Calculate P(math $\cap$ chemistry)

   d) Calculate the probability of selecting a student at random who was either in his math class or his chemistry class, but not both.

   e) Calculate P(Jack's Class^c)

Day 9: Experimental and Theoretical Probability

Warm-Up:

1. A standard six-sided dice is rolled. Find the probability:
    a. Of rolling a number greater than or equal to one.
    b. Of rolling a number greater than six.

2. Calculate the following values.
    a. For club presentations, in how many ways can you schedule 4 speakers out of 12 people to speak?

    b. You pick 4 club members out of 35 to take on a trip. How many different groups are possible?

    c. A wing place 15 flavors at 5 heat levels of boneless and regular wings. How many options do they offer?

3. A customer can pick from the following mix-ins at a frozen yogurt stand: chocolate chips, strawberries, crushed cookies, chopped nuts, sprinkles, & cookie dough. How many ways are there to make a yogurt with no more than three condiments?

4. The counselors at a summer camp are juniors and seniors, with 55% of them male. Of the females, 60% are juniors. Of the males, 56% are seniors.

    a. Create a tree diagram for the scenario, labeling all probabilities and possibilities.

    b. Find P(junior)

    c. If the counselor is a female, find the probability that she is a senior.

    d. Find P(male | junior)

Notes Day 9: Experimental and Theoretical Probability

Part A: Theoretical Probability

Probability is the chance or likelihood of an event occurring. We will study two types of probability, theoretical and experimental.

Theoretical Probability: the ratio of the number of favorable outcomes to the total possible outcomes.

$$P(\text{Event}) = \frac{\text{Number or favorable outcomes}}{\text{Total possible outcomes}}$$

Sample Space: The set of all possible outcomes. For example, the sample space of tossing a coin is {Heads, Tails} because these are the only two possible outcomes. Theoretical probability is based on the set of all possible outcomes, or the sample space.

1. List the sample space for rolling a six-sided die (remember you are listing a set, so you should use brackets {}):

   Find the following probabilities:

   P(2) P(3 or 6) P(odd) P(not a 4)

   P(1, 2, 3, 4, 5, or 6) P(8)

Part B: Experimental Probability

Experimental Probability: the ratio of the number of times the event occurs to the total number of trials.

$$P(\text{Event}) = \frac{\text{Number or times the event occurs}}{\text{Total number of trials}}$$

1. Do you think that theoretical and experimental probabilities will be the same for a certain event occurring? Explain your answer.

2. Roll a six-sided die and record the number on the die in the table below. Repeat this 9 more times

Number on Die	Tally	Frequency
1		
2		
3		
4		
5		
6		
Total		10

Based on your data, find the following experimental probabilities:

P(2) P(3 or 6)

P(odd) P(not a 4)

How do these compare to the theoretical probabilities in Part A? Why do you think they are the same or different?

Why can there be differences between experimental and theoretical probabilities in general?

Part C: Which one do I use?

Fairness:

For a game to be fair, the theoretical probability for each player winning should be equal.

Why should we base our decision about fairness of a game on theoretical probability not experimental probability?

In general, which one to use….

So when do we use theoretical probability or experimental probability? Theoretical probability is always the best choice, when it can be calculated. But sometimes it is not possible to calculate theoretical probabilities because we cannot possible know all of the possible outcomes. In these cases, experimental probability is appropriate. For example, if we wanted to calculate the probability of a student in the class having green as his or her favorite color, we could not use theoretical probability. We would have to collect data on the favorite colors of each member of the class and use experimental probability.

Determine whether theoretical or experimental probability would be appropriate for each of the following.

Explain your reasoning:

1. What is the probability of someone tripping on the stairs today between first and second periods?

2. What is the probability of rolling a 3 on a six-sided die, then tossing a coin and getting a head?

3. What is the probability that a student will get 4 of 5 true false questions correct on a quiz?

4. What is the probability that a student is wearing exactly four buttons on his or her clothing today?

Trying a Game. Is it fair?

5. Jennifer and Jamie are playing a game that involves emptying marbles from two separate bags. The first person to empty their bag wins the game. A spinner with the numbers #4-8 is spun two times and the outcomes are added together. If the sum of the spins is **8,9,10,14,15,16 Jennifer** can remove one marble from her bag. If the sum of the spins is **11,12,13 Jamie** can remove one marble from his bag. To begin the game, each person's bag has 25 marbles.

First, Create a sample space:

Is this a fair game? Why or why not?

Day 10: Experimental vs. Theoretical Probability Lab

Warm-Up:

1. What is the probability of getting Tails first or last or both, in 3 tosses of a fair coin?
(hint: draw a tree diagram for flipping a coin 3 times)

2. A month is chosen from a year. What is the probability of choosing a month that starts with a J or has 30 days?

3. In the Math Club, 7 of the 20 girls are freshman, and 4 of the 14 boys are freshman. What is the probability of randomly selecting a boy or a freshman to represent the Math Club at a statewide math contest?

4. Sandra has 8 pairs of winter gloves in a drawer. 3 pairs of the gloves are fleece, 4 pairs are leather and 1 pair is wool. If Sandra pulls out a pair of leather gloves, what's the probability the next glove she grabs will be wool?

Day 11: Probability Review

Warm-Up:

1. There are 3 quarters, 7 dimes, 13 nickels, and 27 pennies in Jonah's piggy bank. If Jonah chooses 2 of the coins at random,
   a) what is the probability that the first coin chosen is a penny and the second coin chosen is a dime? The first coin is not replaced.
   b) what is the probability that he chooses a quarter and a dime?

2. Given a standard deck of cards, find P(Ace of Spades | black card).

3. Suppose 60% of all teenagers like to watch horror movies. 28% of teenagers that watch horror movie, watch movies in the dark. 76% of teenagers that do not watch horror films, watch movies with the lights on.
   a) Create a tree diagram.
   b) What is the probability that a teenager watches movies with the lights on?
   c) Find P(Dark | Watches Horror Movies)
   d) If the teenagers watch movies in the dark, what is the probability that they do not watch horror films?