Unit 5 Day 6
Law of Cosines

Warm-up

Happiness begins where selfishness ends. – John Wooden

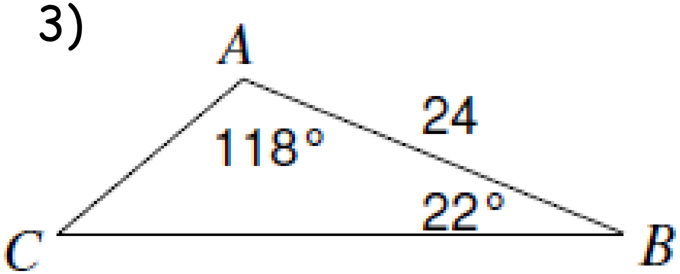
Solve each proportion:

1) $\frac{2x - 3}{3} = \frac{10 - 4x}{2}$ **x = 2.25**

2) $\frac{x + 3}{x + 2} = \frac{x - 1}{x - 4}$ **x = -5**

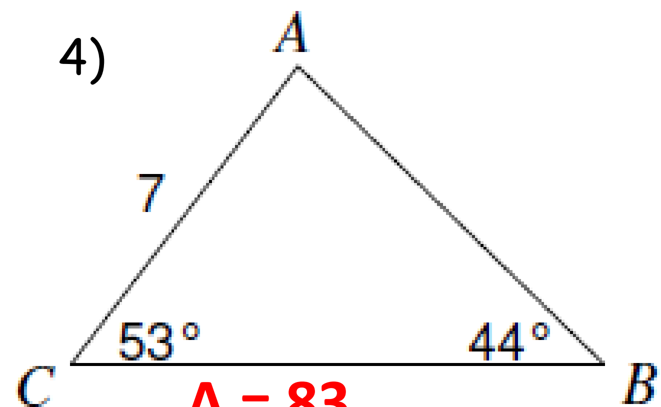
Solve each triangle using Law of Sines. Round to the nearest hundredth.

3)



C = 40
a = 32.97
b = 13.99

4)



A = 83
a = 10
c = 8.05

Warm-up ANSWERS & Work

Happiness begins where selfishness ends. - John Wooden

Solve each proportion:

1) $\frac{2x - 3}{3} = \frac{10 - 4x}{2}$

$$2(2x-3)=3(10-4x)$$

$$4x-6=30-12x$$

$$16x=36$$

$$x=2.25\ or\ \frac{9}{4}$$

x = 2.25

2) $\frac{x + 3}{x + 2} = \frac{x - 1}{x - 4}$

$$(x+3)(x-4)=(x+2)(x-1)$$

$$x^2-x-12=x^2+x-2$$

$$-10=2x$$

$$x=-5$$

x = -5

Warm-up ANSWERS & Work

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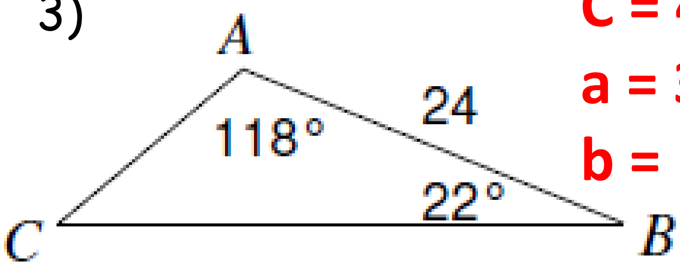
Solve each triangle using Law of Sines.

3)



C = 40
a = 32.97
b = 13.99

1) $180 - 118 - 22 = 40$

$C = 40^{\circ}$

2) $\dfrac{\sin(40)}{24} = \dfrac{\sin(118)}{a}$

$a = \dfrac{24\sin(18)}{\sin(40)}$

$a = 33.0$

3) $\dfrac{\sin(22)}{b} = \dfrac{\sin(40)}{24}$

$b = \dfrac{24\sin(22)}{\sin(40)}$

$b = 14.0$

Warm-up ANSWERS & Work

Happiness begins where selfishness ends. -John Wooden

Solve each triangle using Law of Sines.

1) $$\frac{\sin(44)}{7} = \frac{\sin(53)}{c}$$

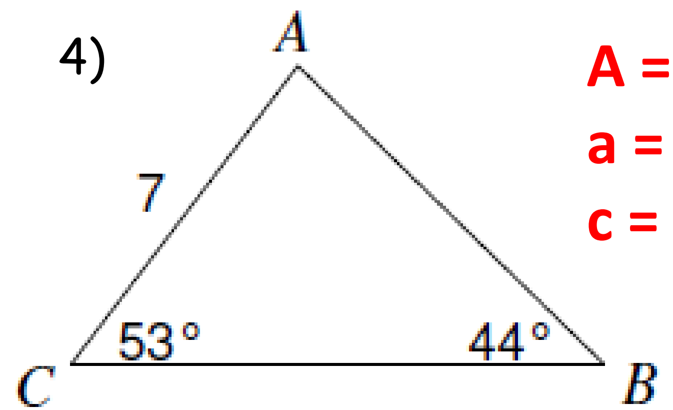
$$c = \frac{7\sin(53)}{\sin(44)}$$

$$c = 8.0$$

2) $$180 - 44 - 53 = 83$$

$$A = 83^\circ$$

4)



A = 83
a = 10
c = 8.05

3) $$\frac{\sin(83)}{a} = \frac{\sin(44)}{7}$$

$$a = \frac{7\sin(83)}{\sin(44)}$$

$$a = 10.0$$

HW Packet p. 9

6. **One Solution**

54°
63 °
44 units

7. **Two Solutions**

Case-1	Case-2
43.7°	136.3°
102.3 °	9.7 °
148.5 units	25.6 units

8. **One Solution**

23°
129°
248.3 units

9. **One Solution**

20.2°
34.8°
62.7 units

HW Packet p. 9

10. **Two Solutions**

Case-1	Case-2
53.8°	126.2°
93.2 °	20.8 °
49.4 units	17.6 units

12. **Two Solutions**

Case-1	Case-2
17.5°	162.5°
146.5 °	1.5 °
22.03 units	1.04 units

11. **One Solution**

R = 48.6°

Q = 42.4°

q = 10.8

13. **One Solution**

(Be careful… find the last angle 1st, then you can solve it ☺)

R = 67°, p = 3.7, q = 15.0

Homework Information!

Packet:

Page 10 (circled problems only)

Page 11 (Read Only)

Page 12 Odds & #18

HW Page 13 and 14 are **extra practice**, if you'd like ☺

Suggestion Of The Day:

Study for Unit 5 Quiz 1

Also, make notes cards of The Law of Sines Formula, Area Of A Triangle with Sine, Trig Function Ratios, AND Law of Cosines. Utilize Website resources to help you study!!

Law of Cosines

THE LAW OF COSINES Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

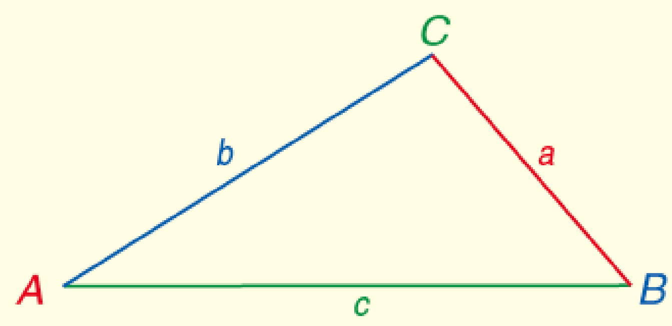
Key Concept — *Law of Cosines*

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then the following equations are true.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

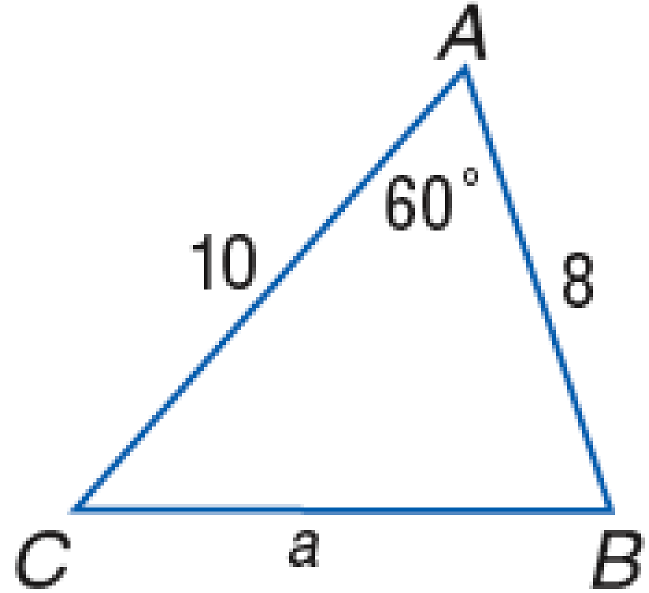


Remember that Law of Sines is used when you have an angle and side across from each other (Remember that sometimes we can subtract from 180 and get and angle and side across from each other)

If you do NOT have an angle and side across from each other, use Law of Cosine.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ex. 1 Use Law of Cosines to find *a*.



Notice this arrangement is SAS.
→ We CANNOT use Law of Sines.

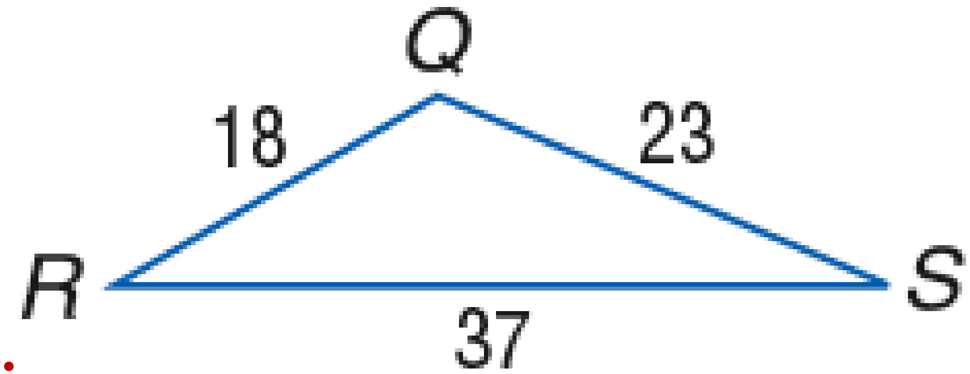
$$a^2 = b^2 + c^2 - 2bc\text{Cos}A$$
$$a^2 = 10^2 + 8^2 - 2(10)(8)\text{Cos}(60)$$
$$a^2 = 164 - 160\text{Cos}(60)$$
$$a = \sqrt{164 - 160\cos(60)}$$
$$a = \sqrt{84} = 2\sqrt{21} \approx 9.2$$

Be sure to keep the back chunk all together. The values in that term are side-by-side so they are multiplied together!

When using Law of Cosines, be careful with signs and with the order of operations!! PEMDAS!! ☺

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ex. 2 Use Law of Cosines to find $m\angle R$.



Notice this arrangement is SSS.
→ We CANNOT use Law of Sines.

$$r^2 = s^2 + q^2 - 2sq\text{Cos}R$$

$$23^2 = 37^2 + 18^2 - 2(37)(18)\text{Cos}(R)$$

$$529 = 1693 - 1332\text{Cos}(R)$$

$$-1164 = -1332\cos R$$

$$R = \cos^{-1}(-1164/-1332)$$

$$R = 29.1^\circ$$

Keep the back term all together. Isolate the back term because it is where the variable is located. Isolate cosR , then do inverse cosine to find R.

When using Law of Cosines, be careful with signs and with the order of operations!! PEMDAS!! ☺

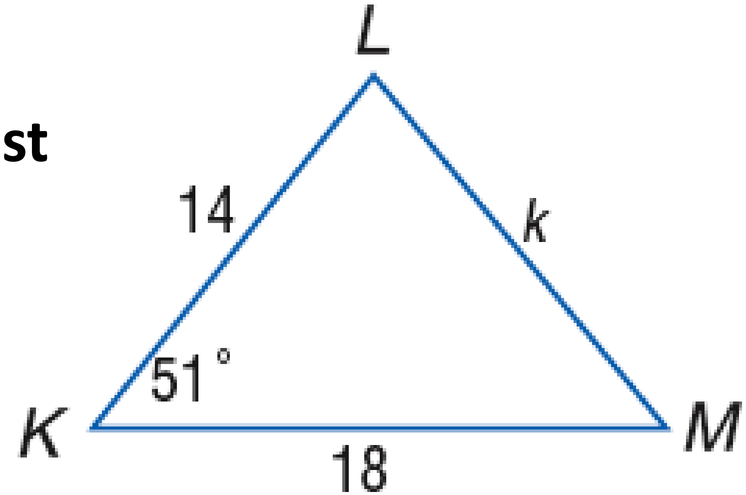
$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Ex. 3

Solve ΔKLM. Round angle measure to the nearest degree and side measure to the nearest tenth.



Tips for SAS steps: You'll need 3 steps just as you did with Law of Sines Problems

1)Think BIG – use law of **Cosines** with the GIVEN angle first

2)Short Side Second with Law of **Sines**

3)Subtract angles from 180°

1st) $k^2 = m^2 + \ell^2 - 2(m)(\ell)\text{Cos}(K)$

$k^2 = 14^2 + 18^2 - 2(14)(18)\text{Cos}(51)$

$$k = \sqrt{196 + 324 - 2(14)(18)\text{Cos}(51)}$$

$$k = 14.2$$

3rd) L = 180 – 51 – 50

L = 79°

2nd) m is the shorter of the given sides, so use it next

$$\frac{Sin(51)}{14.2} = \frac{Sin(M)}{14}$$

$$Sin^{-1}\left(\frac{14 \bullet Sin(51)}{14.2}\right)$$

$$M \approx 50°$$

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Ex. 4: Solving given SSS

Solve ΔABC if a=8, b=10, and c=5.

Remember, if no picture is given, draw one! ☺

Tips for SSS steps: You'll need 3 steps just as you did with Law of Sines Problems

1)Think BIG – use law of Cosines with BIGGEST side first

2)Short Side Second with Law of Sines

3)Subtract angles from 180°

1st) $b^2 = a^2 + c^2 - 2(a)(c)\text{Cos}(B)$

$10^2 = 5^2 + 8^2 - 2(5)(8)\text{Cos}(B)$

$$100 = 25 + 64 - 80\cos(B)$$

$$11 = -80\cos(B)$$

$$-.1375 = \cos(B)$$

$$\cos^{-1}(-.1375) = B$$

$$B = 97.9°$$

2nd) c is the shorter of the given sides, so use it next

$$\frac{Sin(97.9)}{10} = \frac{Sin(C)}{5}$$

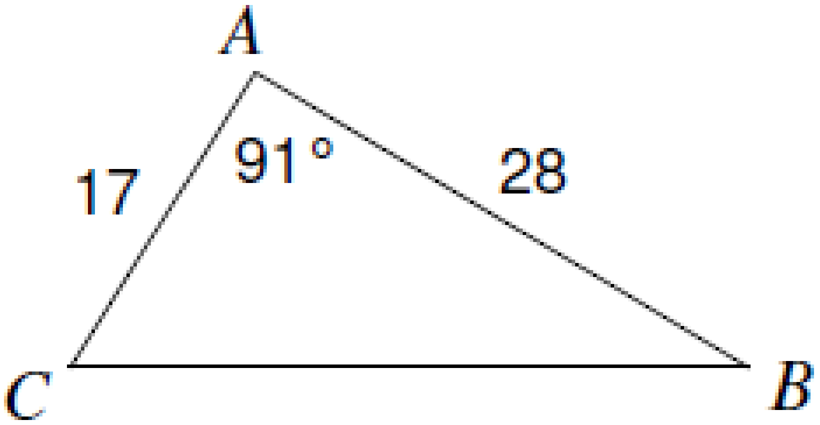
$$C = Sin^{-1}\left(\frac{5 \bullet Sin(97.9)}{10}\right) \approx 29.67°$$

3rd) L = 180 – 97.9 – 29.67

L = 52.4°

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ex. 5 YOU TRY!
Solve the triangle.
Round to the nearest integer.



a = 33 , m∠B= 31° , m∠C= 58°

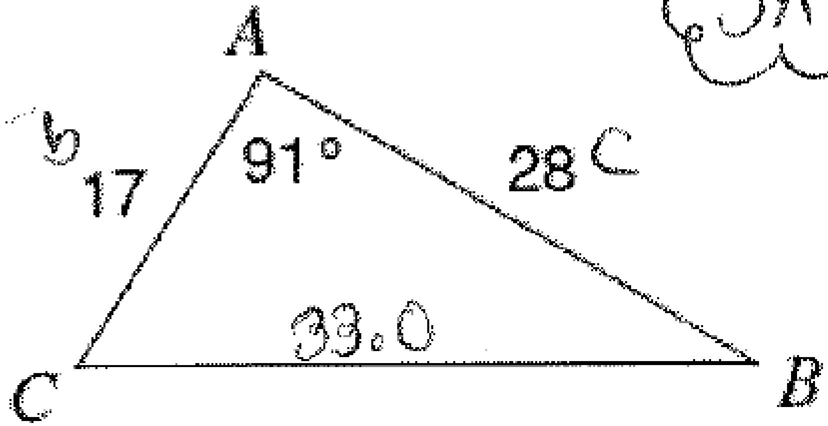
Work Shown on next slide! ☺

Ex. 5 YOU TRY!

Solve the triangle.

Work Shown Below! ☺

SAS



Think big

① $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 17^2 + 28^2 - 2(17)(28)\cos(91)$

$\sqrt{a^2} = \sqrt{1089.61}$

$\boxed{a = 33.0}$

② $\dfrac{\sin(91)}{33.0} = \dfrac{\sin B}{17}$

Short side 2nd

$\sin^{-1}\left(\dfrac{17\sin(91)}{33.0}\right) = B$

$\boxed{B = 31°}$

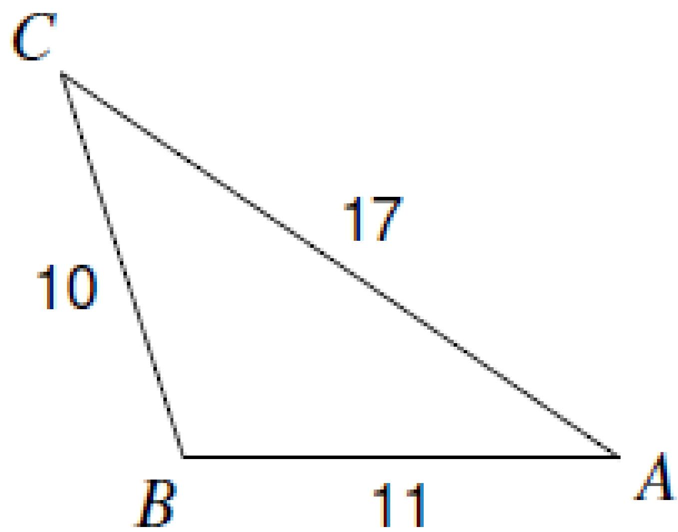
③ subtract from 180°

$$\begin{array}{rl} 180 & \\ -31 & B \\ -91 & A \\ \hline \end{array}$$

$\boxed{58° = C}$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Ex. 6 YOU TRY!
Solve the triangle.
Round to the nearest integer.



m∠A = 34º , m∠B = 108º , m∠C = 38º

Work Shown on next slide! ☺

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

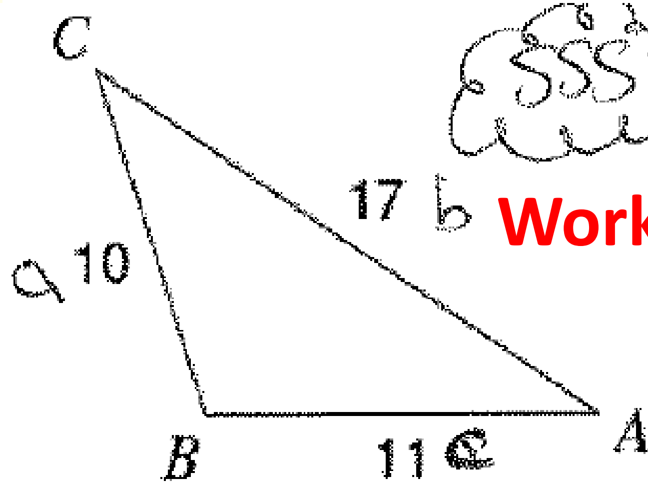
$c^2 = a^2 + b^2 - 2ab\cos C$

SSS

Ex. 6 **YOU TRY!**

Solve the triangle.

Work Shown Below! ☺



Think Big 1st

① $b^2 = a^2 + c^2 - 2ac\cos B$

$17^2 = 10^2 + 11^2 - 2(10)(11)\cos B$

$-10^2 \quad -10^2 \quad -11^2$

-11^2

$\frac{68}{-220} = \frac{-220\cos B}{-220}$

$\cos^{-1}\left(\frac{-68}{220}\right) = B$

$\boxed{B = 108^\circ}$

$\boxed{C = 38^\circ}$

② $\frac{\sin A}{a} = \frac{\sin B}{b}$

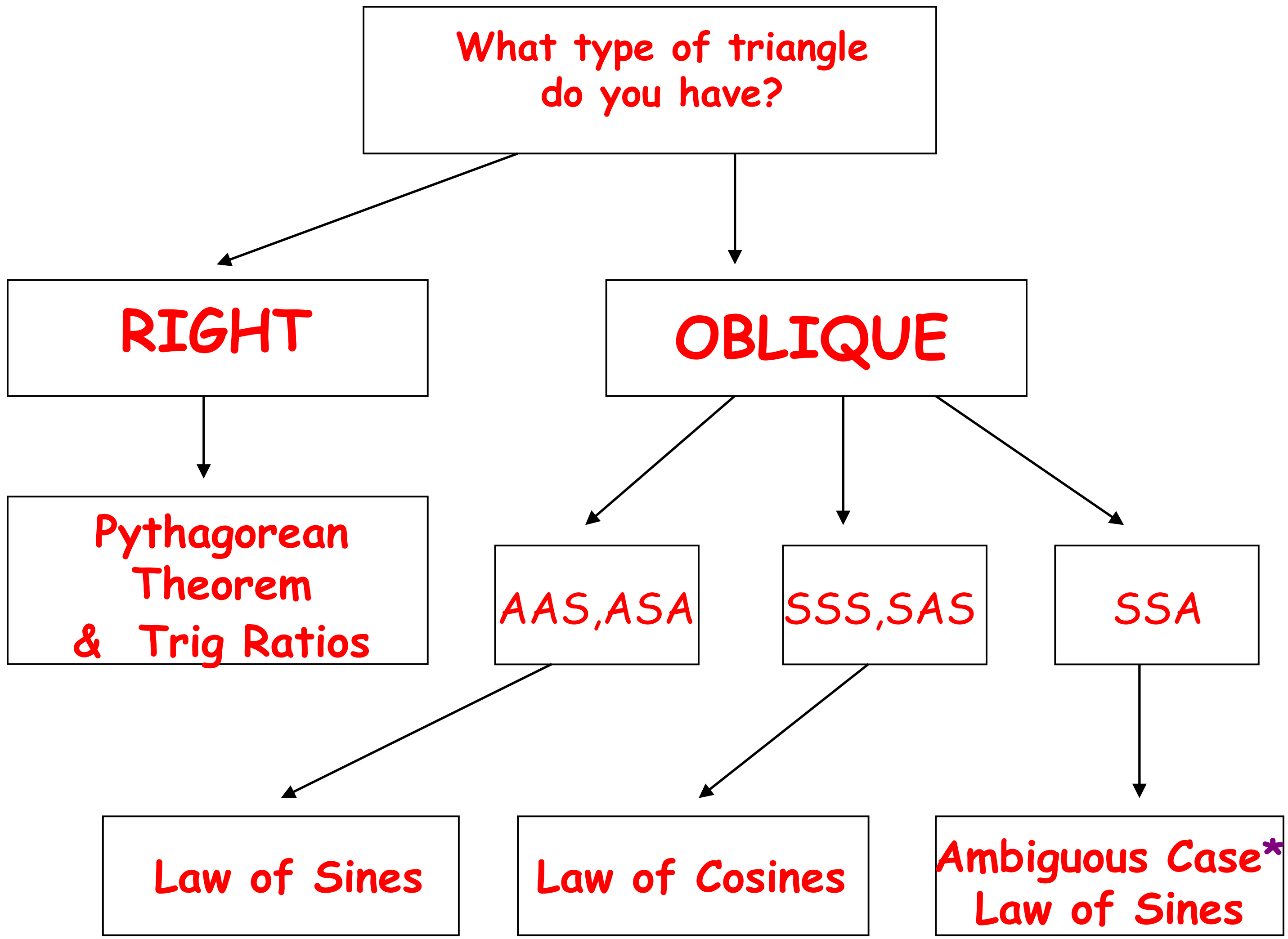
Short Side 2nd

$\frac{\sin A}{10} = \frac{\sin(108)}{17}$

$\sin A = \frac{10\sin(108)}{17}$

$\boxed{A = 34^\circ}$

Which Formula Do I Use?

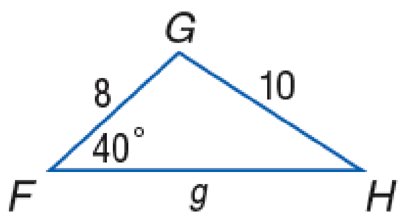


*Remember we have 2 triangles here if the side across from angle is smaller than other side

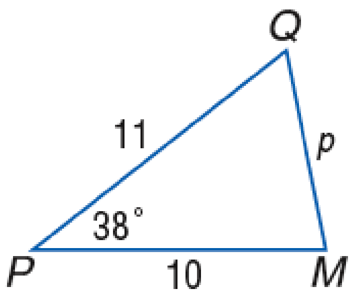
Practice!

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

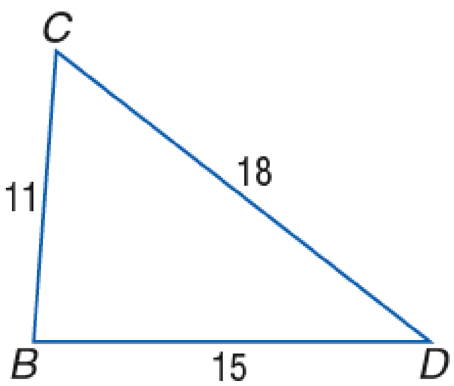
19.



20.



21.



22. $\triangle ABC$: $m\angle A = 42$, $m\angle C = 77$, $c = 6$

23. $\triangle ABC$: $a = 10.3$, $b = 9.5$, $m\angle C = 37$

24. $\triangle ABC$: $a = 15$, $b = 19$, $c = 28$

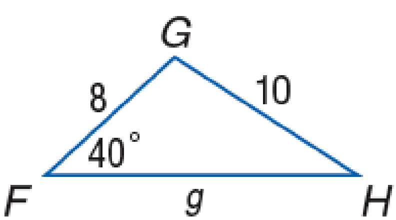
25. $\triangle ABC$: $m\angle A = 53$, $m\angle C = 28$, $c = 14.9$

Practice Answers!

Capital letters are angles, lowercase letters are sides

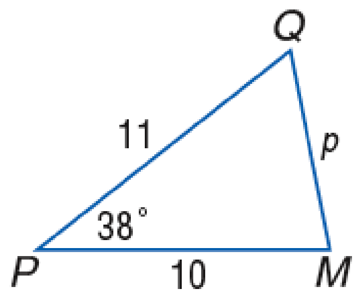
Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

19.



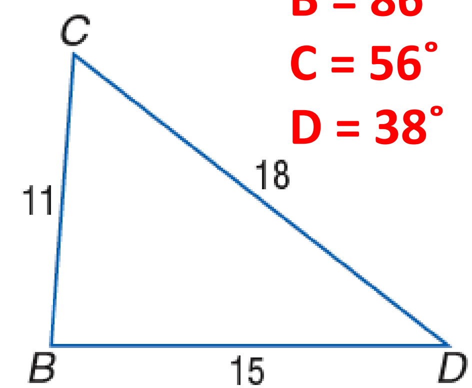
H = 31°
G = 109°
g = 14.7

20.



p = 6.9 M = 79°
Q = 63°

21.



B = 86°
C = 56°
D = 38°

22. $\triangle ABC$: $m\angle A = 42$, $m\angle C = 77$, $c = 6$

23. $\triangle ABC$: $a = 10.3$, $b = 9.5$, $m\angle C = 37$

24. $\triangle ABC$: $a = 15$, $b = 19$, $c = 28$

25. $\triangle ABC$: $m\angle A = 53$, $m\angle C = 28$, $c = 14.9$

22.) B = 61°, b = 5.4, a = 4.1
23.) c = 6.3, A = 80°, B = 63°
24.) A = 30°, B = 39°, C = 111°
25.) B = 99°, b = 31.3, a = 25.3

Puzzle Time!

Law Of Cosines & Law of Sines

What Do You Call A Cow With No Legs?

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Page 10 (circled problems only)
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Page 12 Odds & #18.

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