Unit 5 Day 4:

Law of Sines &
Area of Triangles with Sines

Warm-up:

Draw a picture and solve. Label the picture with numbers and words including the angle of elevation/depression and height/length.

1. A tree 10 meters high cast a 17.3 meter shadow. Find the angle of elevation of the sun.

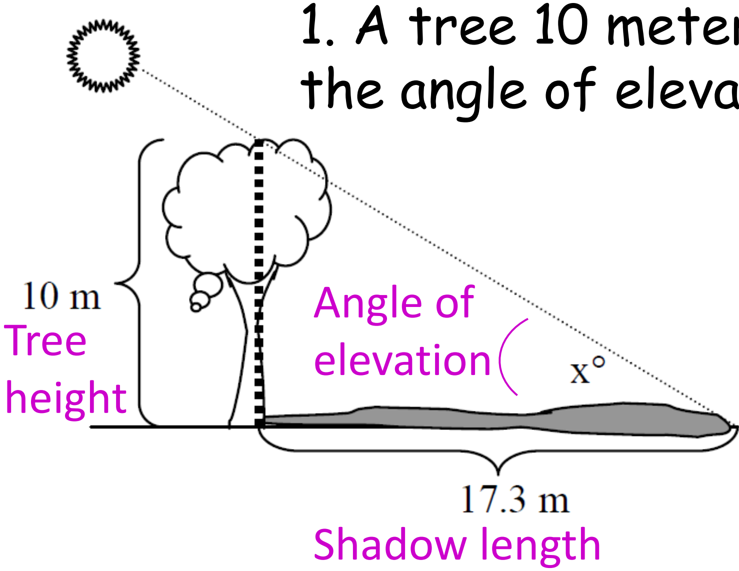
2. A car is traveling up a slight grade with an angle of elevation of 2°. After traveling 1 mile, what is the vertical change in feet? (1 mile = 5280 ft)

3. A person is standing 50 meters from a traffic light. If the angle of elevation from the person's feet to the top of the traffic light is 25°, find the height of the traffic light.

4. Two friends, each 5 foot tall, meet at a 120 foot tall flag pole, then walk in opposite directions. One measures her angle of elevation to the top of the pole to be 38 degrees, while the other friend finds his to be 42 degrees. How far apart are the friends?

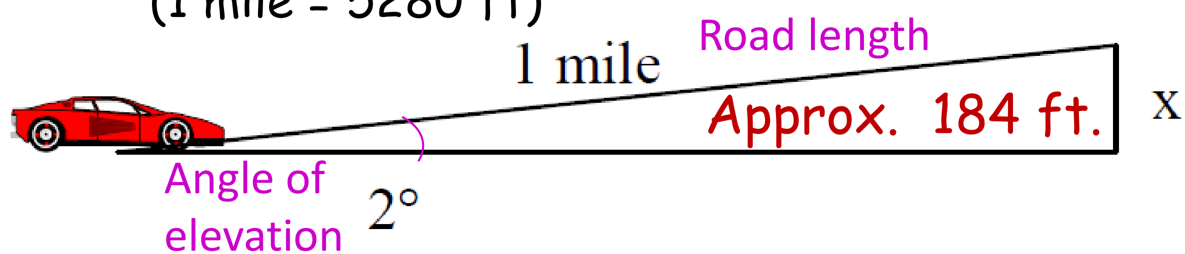
Warm-up Answers

1. A tree 10 meters high cast a 17.3 meter shadow. Find the angle of elevation of the sun.



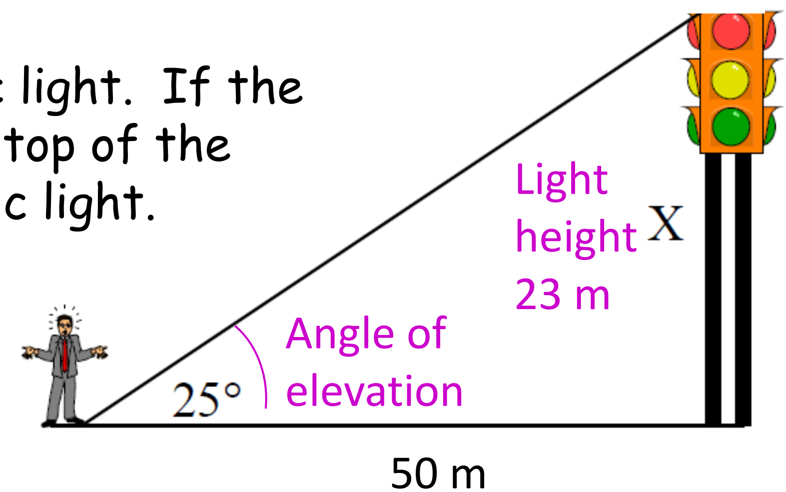
Approx. 30°

2. A car is traveling up a slight grade with an angle of elevation of 2°. After traveling 1 mile, what is the vertical change in feet? (1 mile = 5280 ft)



3. A person is standing 50 meters from a traffic light. If the angle of elevation from the person's feet to the top of the traffic light is 25°, find the height of the traffic light.

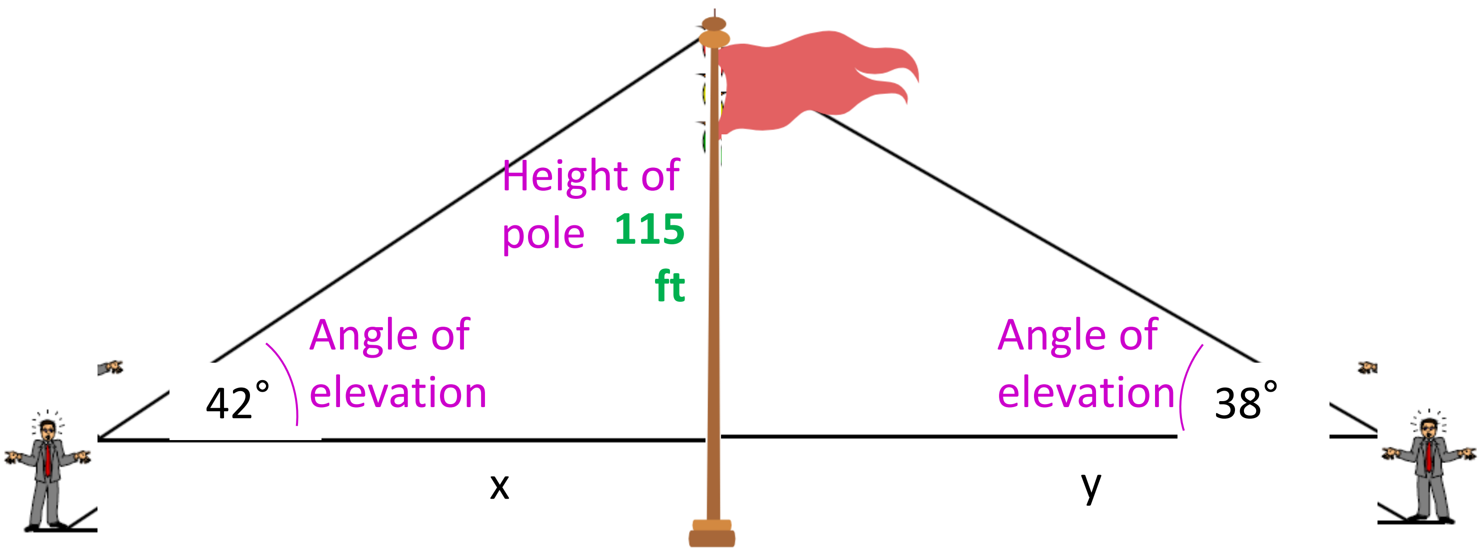
Approx. 23 m



Warm-up Answers

Laws control the lesser man… Right conduct controls the greater one.

4. Two friends, each 5 foot tall, meet at a 120 foot tall flag pole, then walk in opposite directions. One measures her angle of elevation to the top of the pole to be 38 degrees, while the other friend finds his to be 42 degrees. How far apart are the friends?



Approx. 274.9 feet

HW Packet p. 5

1. 7.1 m
2. 8.2 ft
3. 28.6 ft
4. 81.9 m
5. 17.2 ft
6. 57.4ft

HW Packet p. 6

1. 994.3 ft
2. 1664.3 m
3. 29°
4. 29.4 ft
5. 30.1 ft
6. 19.27 ft
7. 82.8°
8. 41.4 m

Homework Information!!

Packet Page:

7 ODDS and 8 ALL

Suggestion Of The Day:

Start Studying For Unit 5 Quiz 1

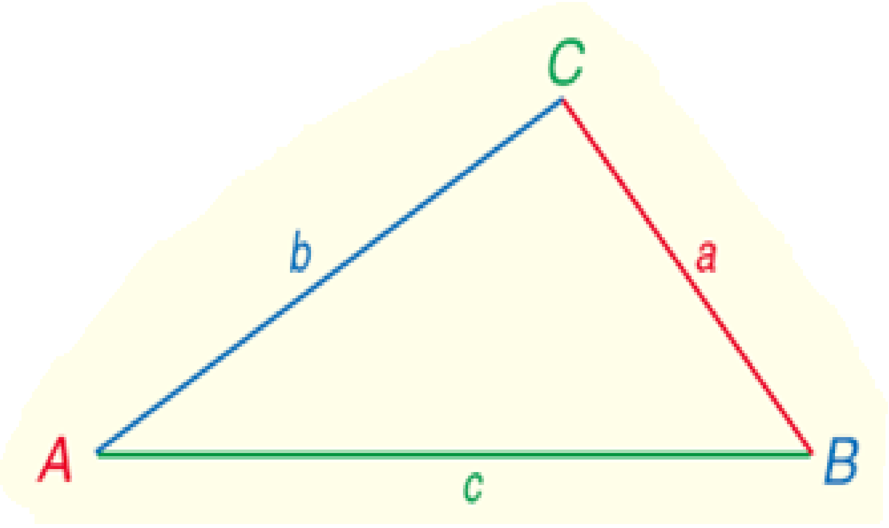
Also, Make Notes Cards Of The Law of Sines Formula, Area Of A Triangle with Sine, And Of Trig Function Ratios

Law of Sines

Note Day 4 Part 1 – Solving Oblique Triangles

- **In trigonometry, the Law of Sines can be used to find missing parts of triangles that are oblique triangles. (Remember an oblique triangle is a non-right triangle.)**

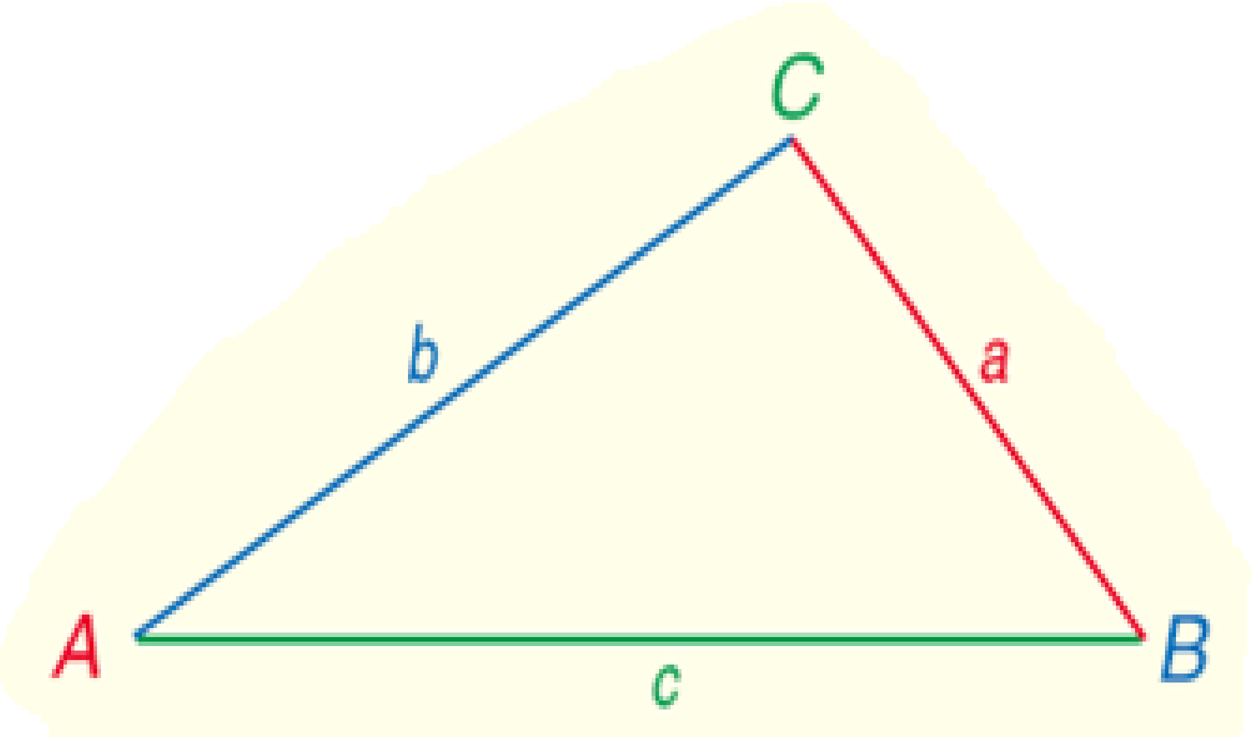
- **In Trigonometry, we use <u>capital letters for angles</u> and <u>lower case letters for sides</u>. We also use special positions for the angles and sides. Angle A is opposite from side a, etc, as shown below.**



Discovery for Solving Oblique Triangles

- **Complete the Discovery**

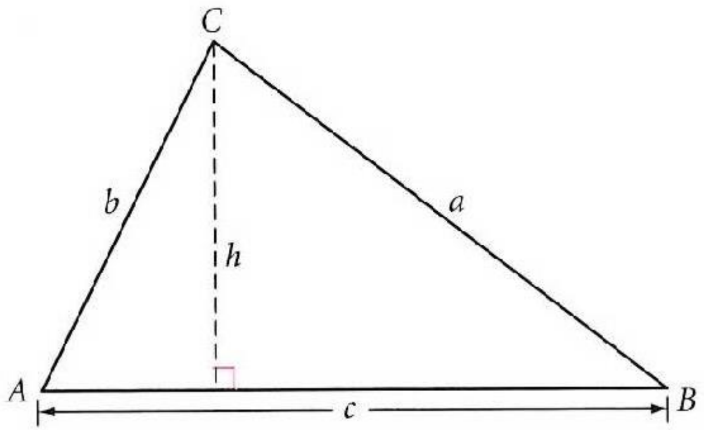
  Notes p. 7 – Complete Tasks 1 Through 8



Discovery for Solving Oblique Triangles!

1. Set up the ratios for **sinA** and **sinB**.

$$\sin A = \frac{h}{b}, \quad \sin B = \frac{h}{a}$$



2. Find ***h*** in terms of ***a*** and the sine of an angle.

$$h = a \sin B$$

3. Find ***h*** in terms of ***b*** and the sine of an angle.

$$h = b \sin A$$

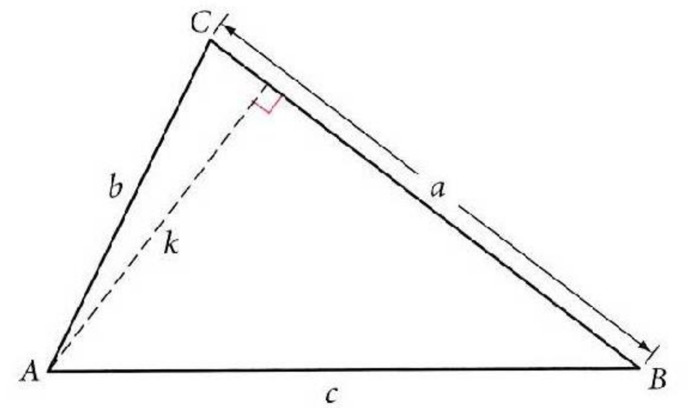
4. Using Algebra, show that $\dfrac{\mathbf{sinA}}{\boldsymbol{a}} = \dfrac{\mathbf{sinB}}{\boldsymbol{b}}$

$$b \sin A = a \sin B$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

5. Find ***k*** in terms of ***c*** and the sine of an angle.

$$\sin B = \frac{k}{c}, \;\; so \;\; k = c\sin B$$



6. Find ***k*** in terms of ***b*** and the sine of an angle.

$$\sin C = \frac{k}{b}, \;\; so \;\; k = b\sin C$$

7. Using Algebra, show that $\dfrac{\mathbf{sinB}}{\boldsymbol{b}} = \dfrac{\mathbf{sinC}}{\boldsymbol{c}}$

$$c\sin B = b\sin C$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

8. Combine steps 4 and 7 to complete the blanks in the following Law of Sines box.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

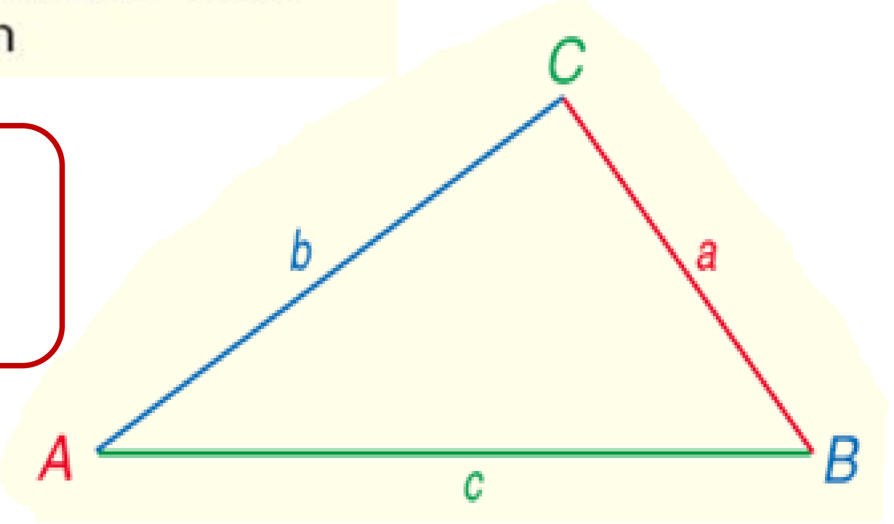
Discussion of Discovery

- Notes p. 7 – Top of Notes p. 8

Law of Sines

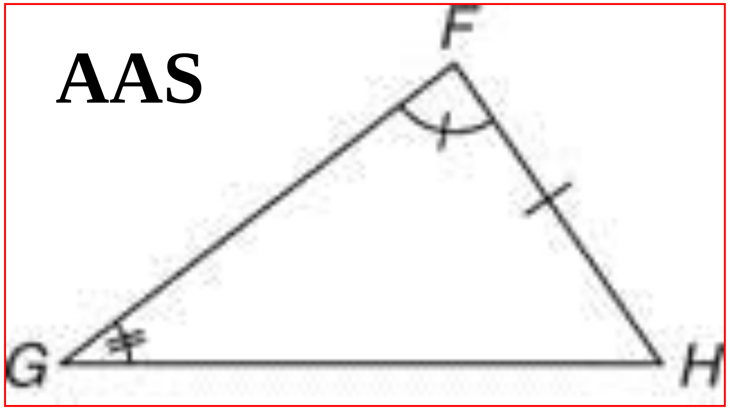
Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then

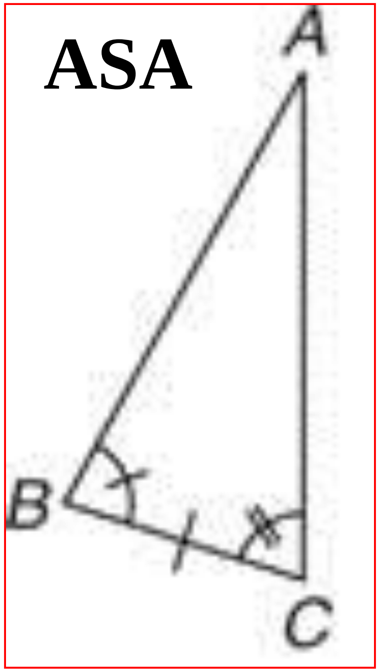
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



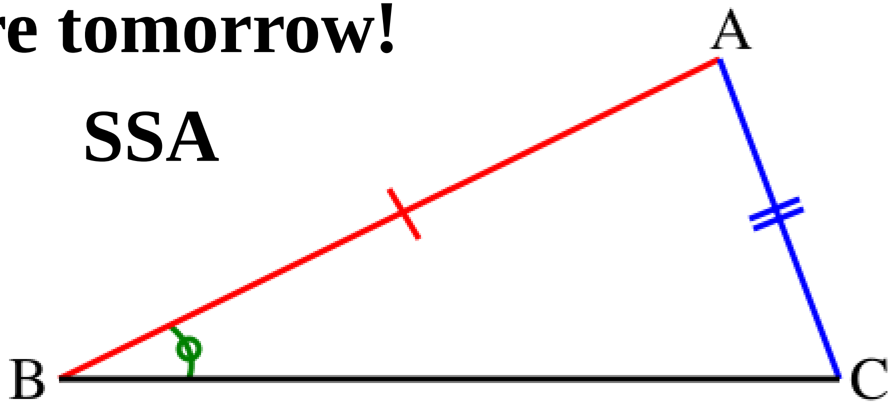
NOTE: to use the Law of Sines, we need an angle and a side across from each other!!

Law of Sines is useful in these cases :



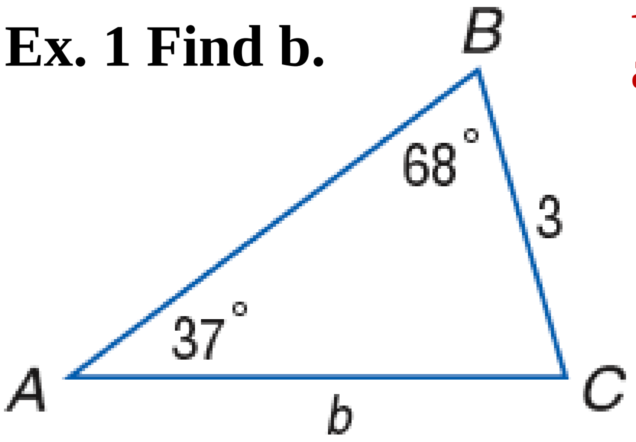


Law of Sines can also be used in this case, but it is ambiguous. We'll discuss this more tomorrow!



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex. 1 Find b.



Pay special attention to how we label the sides and angles.

$$\frac{SinA}{a} = \frac{SinB}{b}$$

$$\frac{Sin(37)}{3} = \frac{Sin(68)}{b}$$

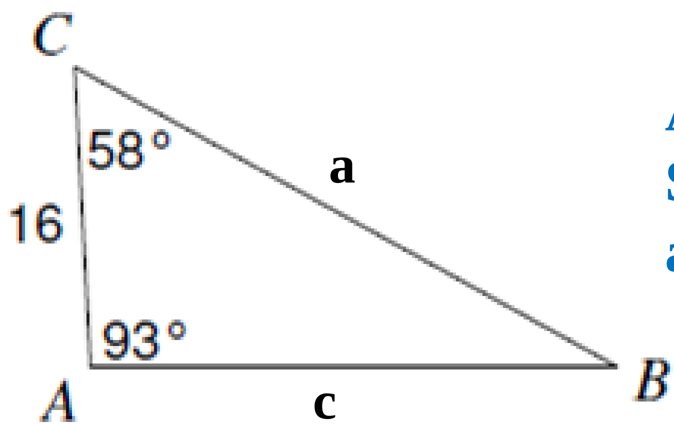
Notice this arrangement is AAS

$$3 \bullet Sin(68) = b \bullet Sin(37)$$

$$b = \frac{3 \bullet Sin(68)}{Sin(37)} \approx 4.6$$

Ex. 2 Find B, a, and c.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



At first, it may appear we can't use Law of Sines because we don't have an angle and side across from each other. BUT could be get one?

$$B = 180 - 93 - 58$$
$$= 29°$$

Notice this arrangement is ASA

$$\frac{SinA}{a} = \frac{SinB}{b}$$

$$\frac{Sin(93)}{a} = \frac{Sin(29)}{16}$$

$$16 \bullet Sin(93) = a \bullet Sin(29)$$

$$a = \frac{16 \bullet Sin(93)}{Sin(29)} \approx 32.96$$

$$\frac{SinB}{b} = \frac{SinC}{c}$$

$$\frac{Sin(29)}{16} = \frac{Sin(58)}{c}$$

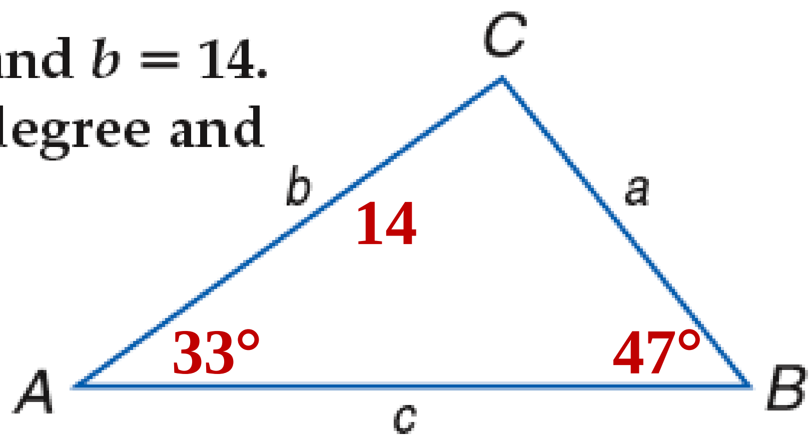
$$16 \bullet Sin(58) = c \bullet Sin(29)$$

$$c = \frac{16 \bullet Sin(58)}{Sin(29)} \approx 27.99$$

The Law of Sines can be used to solve a triangle. Solving a Triangle means finding the measures of all the angles and all the sides of a triangle.

Ex. 3

Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.



First, make a sketch of the triangle and label the parts we know.

Then, find the other parts of the triangle to "SOLVE THE TRIANGLE".

B = 180 – 33 – 47 = 100°

$$\frac{SinA}{a} = \frac{SinB}{b}$$

$$\frac{Sin(33)}{a} = \frac{Sin(47)}{14}$$

$$a = \frac{14 \bullet Sin(33)}{Sin(47)} \approx 10.4$$

$$\frac{SinB}{b} = \frac{SinC}{c}$$

$$\frac{Sin(47)}{14} = \frac{Sin(100)}{c}$$

$$c = \frac{14 \bullet Sin(100)}{Sin(47)} \approx 18.9$$

Indirect Measurement

Ex. 4 Word Problem

When the angle of elevation to the sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow of 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.

Be careful….the 7 degrees takes away from the 90 degree angle shown.

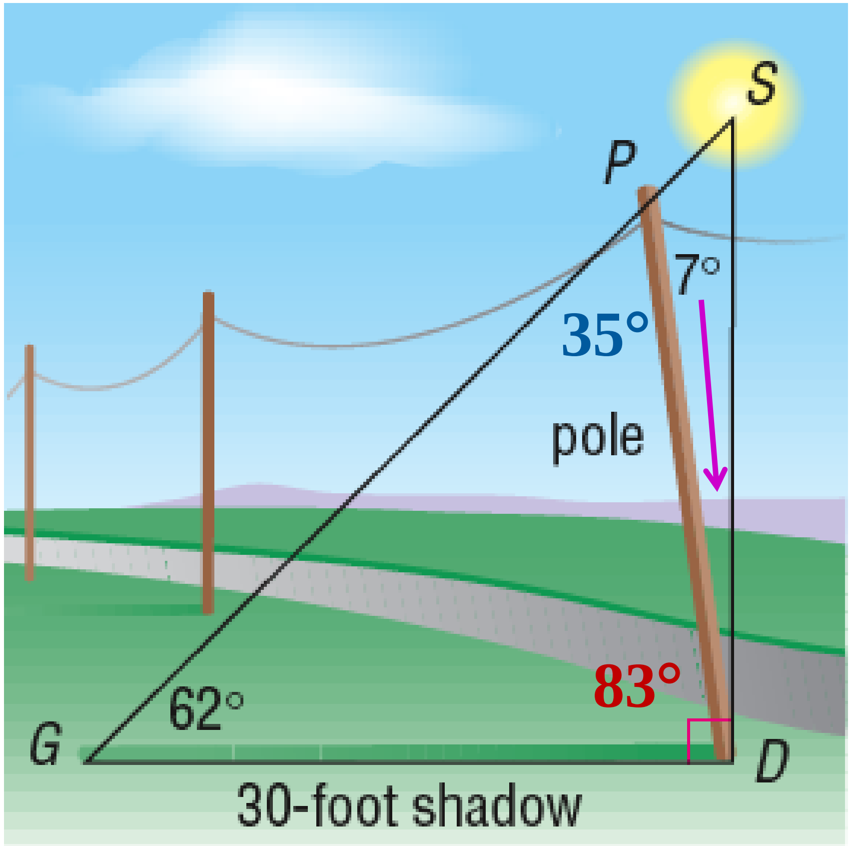
mÐ PDG = 90 – 7 = 83°

We need to find mÐ DPG so that we have an angle and side across from each other!

mÐ DPG = 180 – 83 – 62 = 35°

$$\frac{Sin(35)}{30} = \frac{Sin(62)}{x}$$

$$x = \frac{30 \bullet Sin(62)}{Sin(35)} \approx 46.2\ ft$$



Practice Break!
Try Notes p. 10 #8, 10, 12

Concept Summary — *Law of Sines*

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

Solve each ΔPQR described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

8. $m\angle R = 66, m\angle Q = 59, p = 72$

9. $p = 32, r = 11, m\angle P = 105$ **SSA**

10. $m\angle P = 33, m\angle R = 58, q = 22$

11. $p = 28, q = 22, m\angle P = 120$ **SSA**

12. $m\angle P = 50, m\angle Q = 65, p = 12$

13. $q = 17.2, r = 9.8, m\angle Q = 110.7$ **SSA**

Practice Break Answers!
Try Notes p. 10 #8, 10, 12

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13. $q = 17.2, r = 9.8, m\angle Q = 110.7$ **SSA**

$$8.\ m\angle P = 55^\circ,\ q \approx 75.3,\ r \approx 80.3$$

$$10.\ m\angle Q = 89^\circ,\ p \approx 12.0,\ r \approx 18.7$$

$$12.\ m\angle R = 65^\circ,\ q \approx 14.2,\ r \approx 14.2$$

Notes Part 2:
Finding the Area of Triangles With Sines

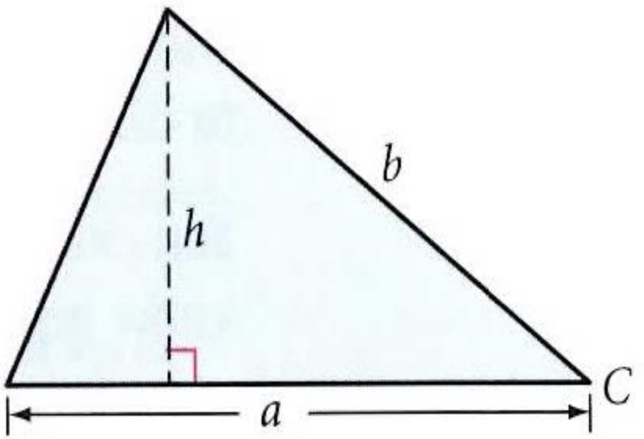
Discovery for Area with Sine

- Complete Bottom of Notes p. 10

Discussion of Discovery

1) What is the formula for the area of a triangle that you remember from other courses?

$$A = \frac{1}{2}bh$$



2) What is formula for the area of this triangle?

$$A = \frac{1}{2}ah$$

3) Find the ratio for sinC.

$$\sin C = \frac{h}{b}$$

4) Suppose we only know the measures of a, b, and angle C – and that we do not know the measure of h. How could we find the area of the triangle? (Hint: use steps 2 & 3)

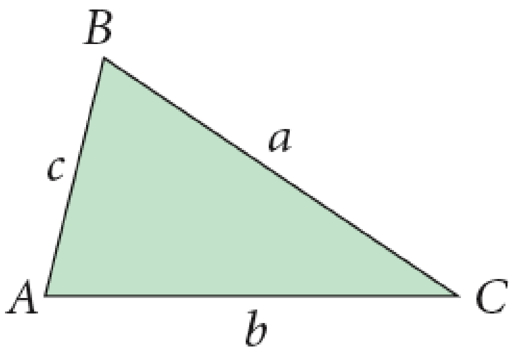
$$\text{Solve for } h,\ h = b\sin C$$

$$A = \frac{1}{2}ab\sin C$$

Theorem 9-1 **Area of a Triangle Given SAS**

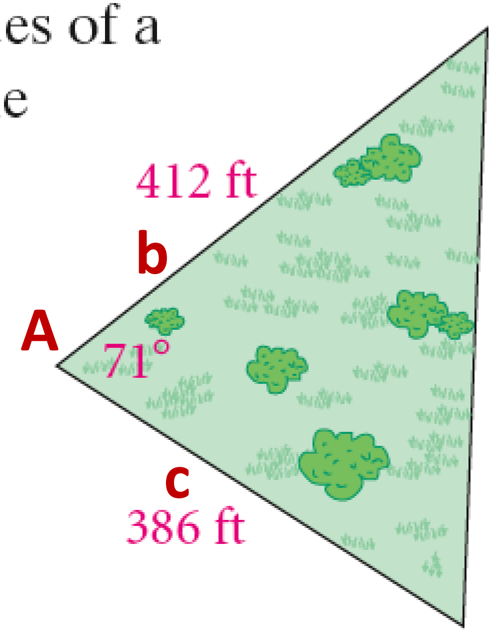
The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.

$$\text{Area of } \triangle ABC = \frac{1}{2}bc(\sin A)$$



Notice you must have the arrangement SAS to use this technique !

Surveying The surveyed lengths of two adjacent sides of a triangular plot of land are 412 ft and 386 ft. The angle between the sides is 71°. Find the area of the plot.



Make sure the triangle is SAS.
Then, label the triangle with b, c, A

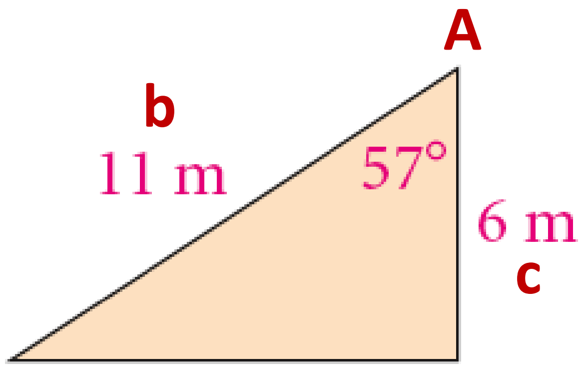
Area = ½ (412 ft)(386 ft) sin(71)
= 75183.9 ft^2

Be sure to use appropriate units!!

One Together!

Find the area of each triangle. Give answers to the nearest tenth.

11.



Does it fit the SAS requirement?

YES! The triangle is SAS.

Then, label the triangle with b, c, A

Area = ½ bc sinA

Area = ½ (11m)(6m) sin(57)

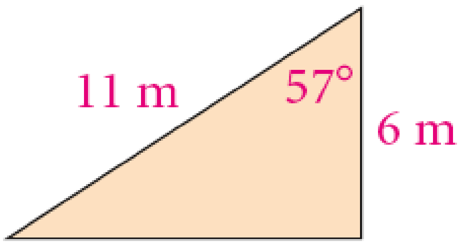
= 27.7 m^2

Be sure to use appropriate units!!

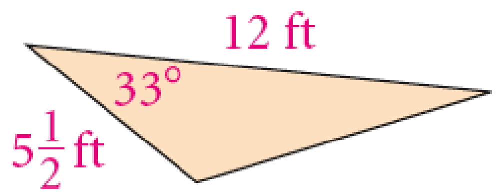
You Try the rest!

Find the area of each triangle. Give answers to the nearest tenth.

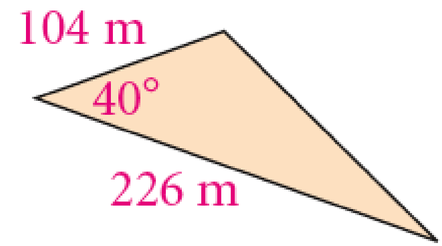
11.



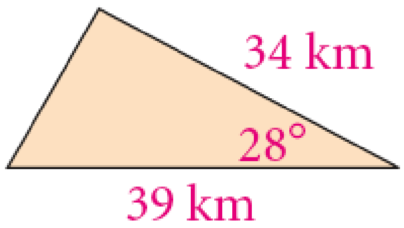
12.



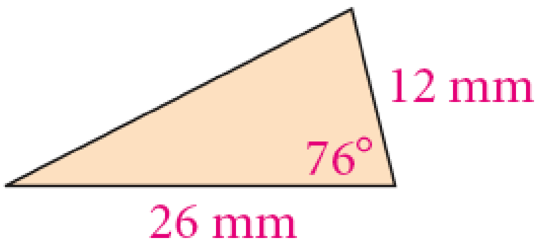
13.



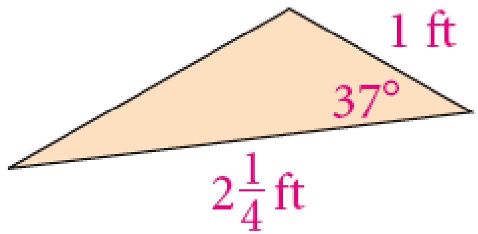
14.



15.



16.

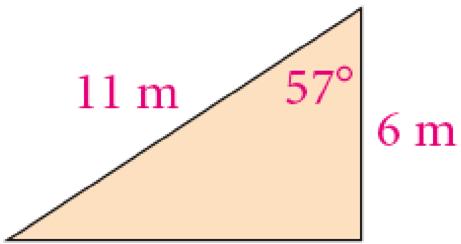


17. **Surveying** A surveyor marks off a triangular parcel of land. One side of the triangle extends 80 yd. A second side of 150 yd forms an angle of 67° with the first side. Determine the area of the parcel of land to the nearest square yard.

You Try Answers!

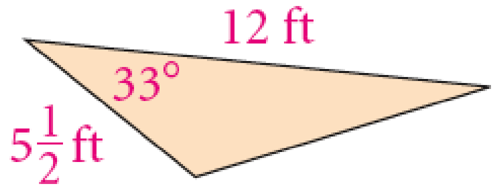
Find the area of each triangle. Give answers to the nearest tenth.

11.



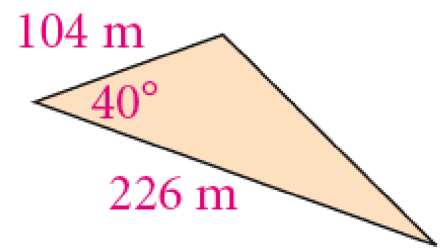
27.7 m²

12.



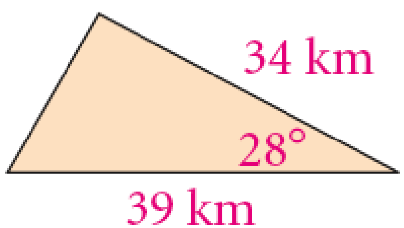
18.0 ft²

13.



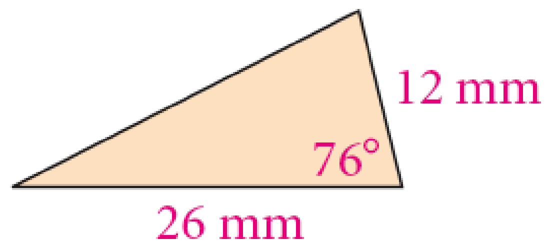
7554.0 m²

14.



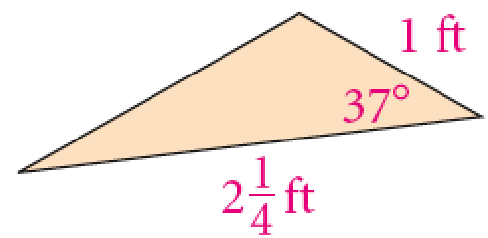
311.3 km²

15.



151.4 mm²

16.



0.7 ft²

17. **Surveying** A surveyor marks off a triangular parcel of land. One side of the triangle extends 80 yd. A second side of 150 yd forms an angle of 67° with the first side. Determine the area of the parcel of land to the nearest square yard.

Hint: Draw a picture first!! **5523 yd²**

Law of Sines Practice!
Try Notes p. 10 #9, 11, 13

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Hint!!

Law of Sines Practice Answers!
Try Notes p. 10 #9, 11, 13

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Hint!!

8. $m\angle P = 55^\circ,\ q \approx 75.3,\ r \approx 80.3$

10. $m\angle Q = 89^\circ,\ p \approx 12.0,\ r \approx 18.7$

12. $m\angle R = 65^\circ,\ q \approx 14.2,\ r \approx 14.2$

9. $m\angle R \approx 19^\circ,\ m\angle Q = 56^\circ,\ q \approx 27.5$

11. $m\angle Q \approx 43^\circ,\ m\angle R \approx 17^\circ,\ r \approx 9.5$

13. $m\angle P \approx 37^\circ,\ p \approx 11.1,\ m\angle R \approx 32^\circ$

Homework Information!!

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7 ODDS and 8 ALL

Suggestion Of The Day:

Start Studying For Unit 5 Quiz 1

Also, Make Notes Cards Of The Law of Sines Formula, Area Of A Triangle with Sine, And Of Trig Function Ratios