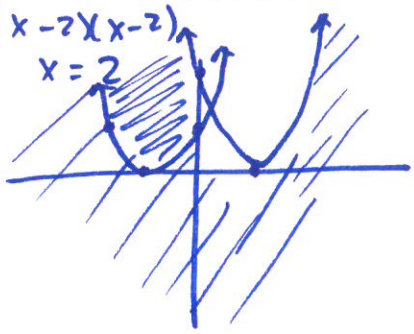
Honors Math 2
HW: ODM Midterm Review

Name KEY

1. On a separate sheet of graph paper, graph $y > x^2 + 2x + 1$ and $y < x^2 - 4x + 4$ and find the intersection.

$(x+1)(x+1)$ $x = -1$ $(x-2)(x-2)$ $x = 2$



2. Solve: $\sqrt{x+2}^2 = x^2$

$x + 2 = x^2$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$ $x = 2$ or ~~-1~~

3. Solve $(x+2)^{3/4} + 3 = 30$

$-3 \quad -3$
$(x+2)^{3/4 \cdot 4/3} = 27^{4/3}$
$x + 2 = 81$
$x = 79$

4. Solve: $\sqrt[3]{3x+1} + 10 = 5$

$\sqrt[3]{3x+1}^3 = -5^3$
$3x + 1 = -125$
$3x = -126$
$x = -42$

5. A painting's original value when purchased was \$350. Six years later, it was worth \$429.20. Find an exponential equation to model the information. Then, find the value of the painting ten years after the purchase. $350 = 429.2b^{0-6}$

$350/429.2 = b^{-6}$
$b \approx 1.0346$

(0, 350) (6, 429.2)
x y x1 y1
$y = 350(1.0346)^x$ $x = 10$
$y =$ \$491.72

6. Find an equation in point ratio form with the points (1, 5) and (4, 30). Round your "b" value to three places. $5 = 30b^{1-4}$

$(5/30)^{-1/3} = (b^{-3})^{-1/3}$
$b \approx 1.8171$

x y x1 y1
$y = 30(1.8171)^{0-4}$
$= 2.75$
$y = 2.75(1.8171)^x$

7. In 2010, you put \$1500 into an account earning 7% annual interest. In what year will the account be worth \$2000?

$2000 = 1500(1.07)^x$
Calc $y_1 = y_2$ $x = 4.25$ years
In 2014

8. Radium has a half-life of 1620 years. Write a function for a 3 mg sample. Find the amount of radium remaining after 50 years. $y = 3(\frac{1}{2})^{x/1620}$

$y = 3(\frac{1}{2})^{50/1620}$
$= 2.94$ mg

9. Solve for x and y: $\left(\frac{3^x}{4^3}\right)^4 = \frac{1}{4^y}$

$x = 0$
$y = 12$

10. Simplify: $\sqrt[4]{10x^7y^3} \cdot \sqrt[4]{60xy^8}$.

$\sqrt[4]{600x^8y^{11}} = x^2y^2\sqrt[4]{600y^3}$

11. Describe how the parabola $y = -(x - 4)^2 + 3$ is shifted from $y = x^2$.

reflect over x-axis, right 4 up 3

12. Sketch the graph of the function on a separate piece of paper. $y = x^2 + 15x + 54$

a. *Find the x-intercepts.* (-9,0) (-6,0) (x+9)(x+6)

b. *Find the axis of symmetry.* x = -7.5

c. *Find the vertex.* (-7.5, -2.25)

d. *Find the y-intercept.* (0, 54)

e. *Is the vertex a max or a min?* min

13. Find the equation of a quadratic function with intercepts at (-2, 0) and (4, 0) and a vertex at (1, 6).

$y = k(x+2)(x-4)$

$6 = k(1+2)(1-4)$

$6 = k(-9)$

$-6/9 = k$ $\quad k = -2/3$

$y = -2/3(x+2)(x-4)$

$y = -2/3(x^2 - 2x - 8)$

$y = -\frac{2}{3}x^2 + \frac{4}{3}x + \frac{16}{3}$

Factor and find the **solutions**.

14. $2v^2 + 11v + 5 = 0$

$2 \cdot 5 = 10$

$10 + 1 = 11$

$(2v^2 + 10v) + (v + 5) = 0$

$2v(v+5) + 1(v+5) = 0 \rightarrow (2v+1)(v+5) = 0$

$2v + 1 = 0$

$2v = -1$

$v = -1/2$

$v = -5$

15. $7a^2 + 53a + 28 = 0$

$(7a^2 + 4a) + (49a + 28) = 0$

$a(7a+4) + 7(7a+4) = 0 \rightarrow (a+7)(7a+4)$

$7a + 4 = 0$

$7a = -4$

$a = -4/7$

$a = 7$

16. $16b^2 + 60b - 100 = 0$

$4(4b^2 + 15b - 25) = 0$

$4(4b^2 + 20b - 5b - 25) = 0$

$4(4b(b+5) - 5(b+5)) = 0$

$4(4b-5)(b+5) = 0$

$4b - 5 = 0$

$4b = 5$

$b = 5/4$

$b = -5$

Find the **discriminant** and tell the number/type of solutions.

17. $b^2 + 16b + 64 = 0$

$16^2 - (4 \cdot 1 \cdot 64) = 0$ — 1 real rational solution

18. $x^2 - 4x + 24 = 0$

$(-4)^2 - (4 \cdot 1 \cdot 24) = -80$ — 2 imaginary solutions

19. $2k^2 + 22k + 60 = 0$

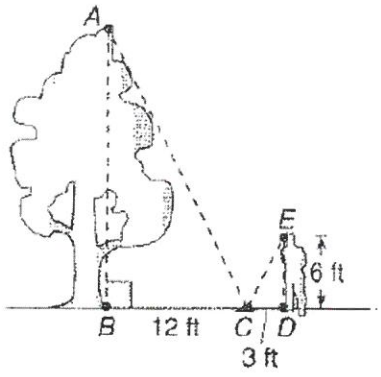
$2(k^2 + 11k + 30)$ $\quad (11)^2 - (4 \cdot 1 \cdot 30) = 1$ — 2 real rational solutions

20. The following function models how much money a certain company makes after a certain amount of time. At what time did they make the least amount of money?

$v(t) = 800 - 28t + .25t^2$ $\quad t = 56$

↳ find x-value of the minimum (vertex)

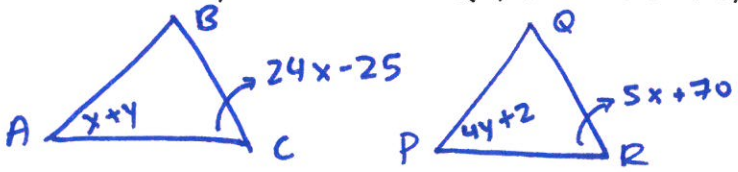
21. It is a law of physics that when a beam of light strikes a mirror, the angle of incidence (∠ACB) and angle of reflection (∠ECD) are equal. So in order to find the height of a tree, you place a mirror 3 feet in front of you on the ground and 12 feet from the base of the tree, as shown. The mirror is placed just so the light from the top of the tree hits the mirror and reflects into your eyes. Find the height of the tree, using a proportion.



$\frac{12}{3} = \frac{x}{6}$ $3x = 72$ $x = 24$ ft

Similar Δ's have ≅ corresponding angles

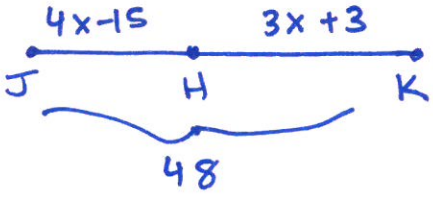
22. Find x and y if $\Delta ABC \sim \Delta PQR$, $m\angle R = 5x + 70$, $m\angle C = 24x - 25$, $m\angle P = 4y + 2$, $m\angle A = x + y$.



$24x - 25 = 5x + 70$
$19x = 95$
$x = 5$

$4y + 2 = x + y$
$4y + 2 = 5 + y$
$3y = 3$
$y = 1$

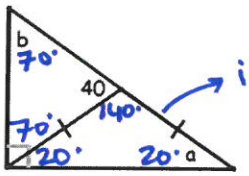
23. Given that H is between J and K, JK = 48, JH = 4x - 15, and HK = 3x + 3, find the value of x, the length of JH, and the length of HK.



$4x - 15 + 3x + 3 = 48$
$7x - 12 = 48$
$7x = 60$
$x = 60/7$ or 8.6

JH = 19.3
HK = 28.7

24. Find a and b in the figure.



isosceles

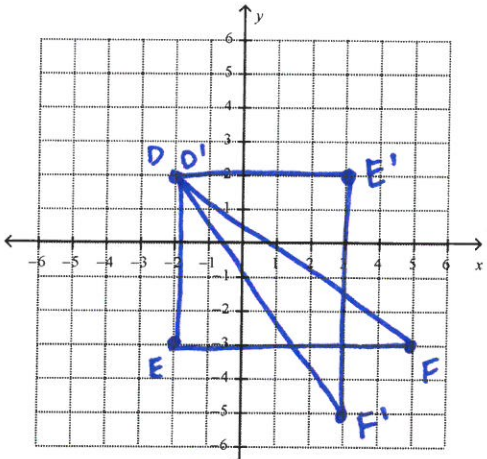
a = 20°
b = 70°

25. The vertices of a triangle are D(-2, 2), E(-2, -3) and F(5, -3). Graph and label the image with a reflection over the line y = –x. Name the image vertices below. $(x, y) \rightarrow (-y, -x)$

D' (-2, 2) E' (3, 2) F' (3, -5)

Write the algebraic rule for a reflection over y = –x.

$(x, y) \rightarrow (-y, -x)$

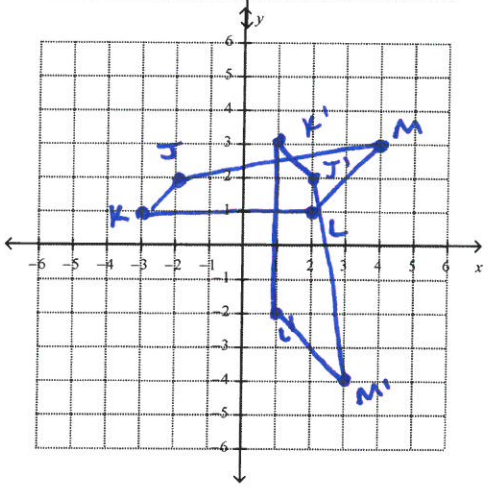


26. Graph and label the points J(-2, 2), K(-3, 1), L(2, 1) and M(4, 3) and then rotate the figure 270°. Graph and label the image points, and write their coordinates below. Then, write the algebraic rule for the transformation.

J' (2, 2) K' (1, 3)
L' (1, -2) M' (3, -4)

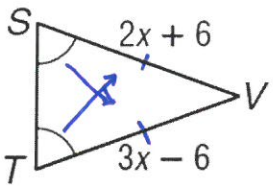
Write the algebraic rule for the rotation 270°:

$(x, y) \rightarrow (y, -x)$



27. Find the value of x.

isosceles



$2x+6 = 3x-6$

$12 = x$

28. Graph ΔABC with A(5, -3), B(2, -4), and C(1, 1), then graph the image of ΔABC after the translation of < 4, 1 > then reflection over the x-axis.

Label all your points then, write the coordinates of the final image below.

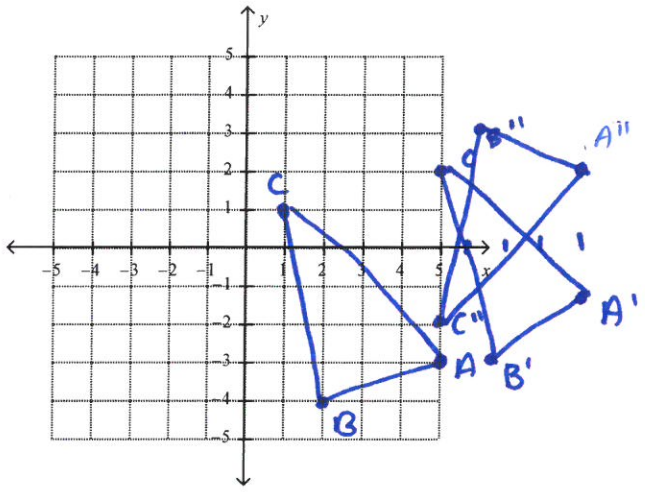
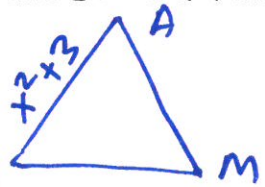


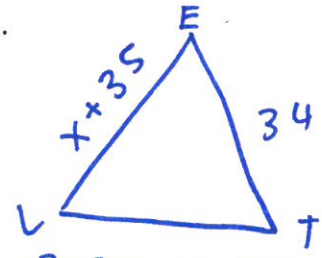
Image A'(9, -2) B'(6, -3) C'(5, 2)

A''(9, 2) B''(6, 3) C''(5, -2)

29. $\Delta SAM \cong \Delta LET$. If $SA = x^2 + 3x$, $LE = x + 35$ and $ET = 34$. Find SA.

$SA = (5)^2 + 3(5) = 40$





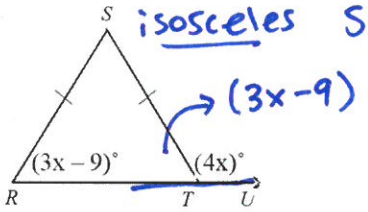
$x^2 + 3x = x + 35$

$x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$x = -7$ or 5

30. Find the value of x.



$3x - 9 + 4x = 180$

$7x = 189$

$x = 27$

31. Solve for x. (hint: use logs)

a. $20^{-6x} + 6 = 55$

$20^{-6x} = 49$

$-6x \log 20 = \log 49$

$x = \frac{\log 49}{-6 \log 20} = -.2165$

b. $\frac{5 \cdot 6^{3x}}{5} = \frac{20}{5}$

$6^{3x} = 4$

$3x \log 6 = \log 4$

$x = \frac{\log 4}{3 \log 6} = .2579$

32. Solve the inequality and write your answer in set notation. $0 \geq 3x^2 - 2x - 5$

$\{x \mid -1 \leq x \leq 5/3\}$ ← check test points

$0 \geq 3x^2 + 3x - 5x - 5$

$0 \geq 3x(x+1) - 5(x+1)$

$(3x-5)(x+1)$

$x = 5/3 \quad x = -1$

33. Solve for x. $0 = x^2 - 4x - 8$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{4 \pm \sqrt{(-4)^2 - 4(1)(-8)}}{2(1)} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3}$

34. Find the domain and range for each function.

a. $f(x) = 3^{x+2} - 4$

Domain: $(-\infty, \infty)$

Range: $(-4, \infty)$

Asymptote: $y = -4$

b. $g(x) = \log(x+7) - 9$

Domain: $(-7, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -7$

Each would have 2 Δ's with corresponding congruent parts

35. 5 Postulates for Congruent Triangles

① SAS 

② ASA 

③ SSS 

④ AAS 

⑤ HL