Direct, Inverse, and Compound Variation

Direct Variation

Direct variation equations are written in the form ____________________, where ______ is the __________________________ (which is also called the constant of proportion).

This equation can also be expressed as _________________.

This form helps us realize that in a direct variation relationship the quotient of y divided by x will always be the constant of variation.

Examples of Formulas that Involve Direct Variation Relationships

- $C = \pi d$; Where _______________ is directly proportional to _______________________, with a constant of variation equal to _________________________.
- $A = \pi r^2$; Where _______________ is directly proportional to _______________________, with a constant of variation equal to _________________________.

Example Problem

James is running at a constant pace. After 4 minutes he has run 700 meters. If he keeps this constant pace, how far will he have run in 9 minutes? Find the constant of variation and write an equation that relates the time that he runs, t, with the distance that he runs, d.

Inverse Variation

Inverse variation equations are written in the form ___________________, where ___is the ____.

This equation can also be expressed as _____________________.

This form helps us realize that in an inverse variation relationship, the product of x and y will always be the constant of variation.

Inverse variation relationships are a type of rational function. The graph of an inverse variation equation is a curve that has two ____________________________. An _________________________ is a line that the graph never touches. The graph of $y = \frac{1}{x}$ has a horizontal asymptote at y=0 and a vertical asymptote at x=0. When x is greater than 0 and is increasing, the graph will curve down and approach y=0. When x is less than 0 and is decreasing the graph will again approach y=0.

Direct, Inverse, and Compound Variation

Sketch the graph of $y = \frac{1}{x}$:

Example Formulas that Involve Inverse Variation Relationships

- $t = \frac{500}{r}$, Where ________________________ is _________________________ proportional to _________________________ with a constant of variation equal to _____________________.
- $I = \frac{1}{d^2}$, Where the intensity of light radiating from a sources varies ________________________ as the square of the distance from the source.

Example Problem

Two different rectangles have the same area. One of the rectangles is 5cm wide and 12cm long. The other rectangle is 6cm wide. What is the length of the other rectangle? Write an equation that relates the length, l, of a rectangle with this area to the width, w, and identify the constant of variation for this relationship.

Compound Variation: Both Inverse and Direct Variation Relationships

- $D = \frac{M}{V}$; Where _________________________ is _______________________ proportional to _________________ and _________________________ proportional to _____________________.
- $I = \frac{V}{R}$; Where _________________________ is _______________________ proportional to ____________________ and ______________________ proportional to _____________________.

Identify which relationships involve direct variation, inverse variation, both, or neither.

1. The formula for the volume of a sphere
2. The equation $R = \frac{V}{I}$
3. The equation $xy = 10$

Write an equation to model each relationship.

4. The variable *a* varies inversely with the variable *b* with a constant of proportionality equal to 7.
5. The variable r varies directly to the square root of s with a constant of proportionality equal to 5.
6. The variable q varies directly to the square of w and inversely to k.